

Technical Report, SC_TR_0010

Structural Modeling Project - Overview

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I. INTRODUCTION

Structural modeling is a powerful set of methods used to discover poorly defined or undefined systems. As indicated by the name, these methods focus on discovering the system structure. The fundamental construct used to discover the system structure is the organizing system relationship. This document presents and discusses the general context for structural modeling as well as its relationship to general systems theory (GST.) Four specific system structuring relationships are identified and discussed in greater detail.

This section concludes with some key definitions and the context within which the authors have pursued this research. Section II, General Context, discusses common features associated with all structural modeling activities. Section III, General Systems Theory (GST), identifies common aspects between GST and structural modeling. Section IV, System Structuring Relationships, provides a closer look at four common system structuring relationships. Section V, Summary and Conclusions, places the current work in context with a closing discussion.

A. Key Definitions

To clarify terms and appropriately express the ideas in this paper, specific definitions for ‘system,’ ‘natural language relationship,’ ‘mathematical relations,’ ‘mathematical model relations,’ ‘Boolean reasoning,’ ‘complexity,’ ‘strict order,’ ‘partial order,’ and ‘degenerative structural thread’ are provided.

A *system* may be defined in a number of ways. This paper uses a ‘construction rule’ definition; that is, a system is a set of two or more objects with a structural relationship (or relationships) mapped over the object set [Simpson and Simpson, April, 2006].

A *natural language relationship* is a term used in human conversation and contextual discourse to indicate some type of order, structure or other manner in which two or more objects are associated between and among themselves. A natural language relationship conveys substantive real-world knowledge, and is an interpretive relationship. Warfield identified six categories of interpretive relationships: 1) definitive, 2) comparative, 3) influence, 4) temporal, 5) spatial, and 6) mathematical [Warfield, 1994:60-61].

Mathematical relations focus mainly on the relations of sets, and set members. Warfield built on Hilbert’s ‘language pair’ concept of metalanguage and object language to view mathematical relations as the ‘object language,’ while viewing natural language relationships as the ‘metalanguage’ [Warfield, 1994:47].

Mathematical model relations are the formal mathematical constructs used to represent the natural language structuring relationships in a well-defined formal manner.

Boolean reasoning is based on Boolean equations, and not on the predicate calculus. Boolean reasoning is based on the Blake canonical form, and syllogistic reasoning. The Boolean reasoning used in this paper is similar to, but different than, switching theory or Boolean minimization approaches [Brown, 2003]. Warfield’s use of a zero (0) to represent either the formal logical notion of false (mathematical concept) or the empirical real-world state of unknown (empirical knowledge), is a key operation that integrates the

operations of the system ‘metalanguage’ (natural language relationships) and system ‘object language’ (mathematical relations).

Complexity is defined as the measure of the difficulty, effort and/or resources required for one system to effectively observe, communicate, and/or interoperate with another system [Simpson and Simpson, 2009, p. 2].

Order: If a binary relation, R , is reflexive and transitive, and R and the complement of R , are antisymmetric, then R is called an order [Simpson and Simpson, 2014].

Partial Order: If the complement of R is not antisymmetric and all other conditions are met, then R is called a partial order [Simpson and Simpson, 2014].

Degenerative structural thread is defined as a malformed or degenerative structural thread that is composed of a single object. Given a single object there is no other object to support a relationship connection.

II. GENERAL CONTEXT

This paper addresses the general approach associated with the solution of large-scale problems and the effective engagement of complexity. Ubiquitous situations involving poorly defined or undefined systems of problems, combined with constrained resources for evaluation and analysis, presents a constant challenge to all individuals that are tasked with addressing problems of this type.

Structural modeling was developed by John N. Warfield to address large-scale sociotechnical problems. This class of problem has a number of common elements which tend to increase the difficulty associated with the identification of an acceptable solution. Many of these common elements are rooted in the uncertainty associated with the problem structure, unknown range of problem solutions, solution approaches and techniques as well as the limited ability of humans to clearly communicate and reason about issues at this scale. The scientific process is a key element in the presented solution process. While the scientific process plays a key role in system solutions, some types of systems are less susceptible to analytical solutions than others. See Figure 1, General System Types [Weinberg, 2001:18].

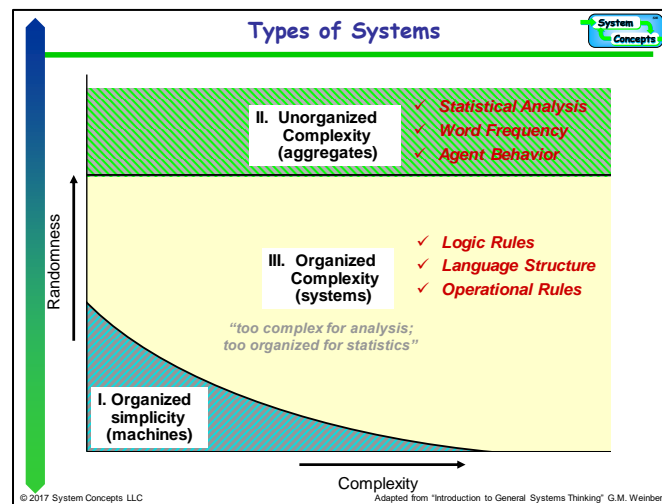


Figure 1. General System Types.

A. Problem Solution Types

Simple analysis may be used on entities with low complexity and low randomness scores. Statistical analysis may be used on entities with a high randomness score where individual component characteristics may be aggregated (the law of large numbers applies in these cases.) System analysis is required in areas where the entity has moderate to high complexity as well as some organizing structure that prevents the application of statistical analysis [Simpson and Simpson, 2005].

Simple analysis can be viewed as the application of known solution patterns applied to known problem inputs that generate a set of known problem outputs. As these problem aspects become more uncertain and contain more unknown components the problem complexity becomes greater. Therefore, the identification of an unknown system structure reduces the complexity associated with any given problem space. Figure 2, Problem – Solution Types, presents a range of problem complexity configurations.

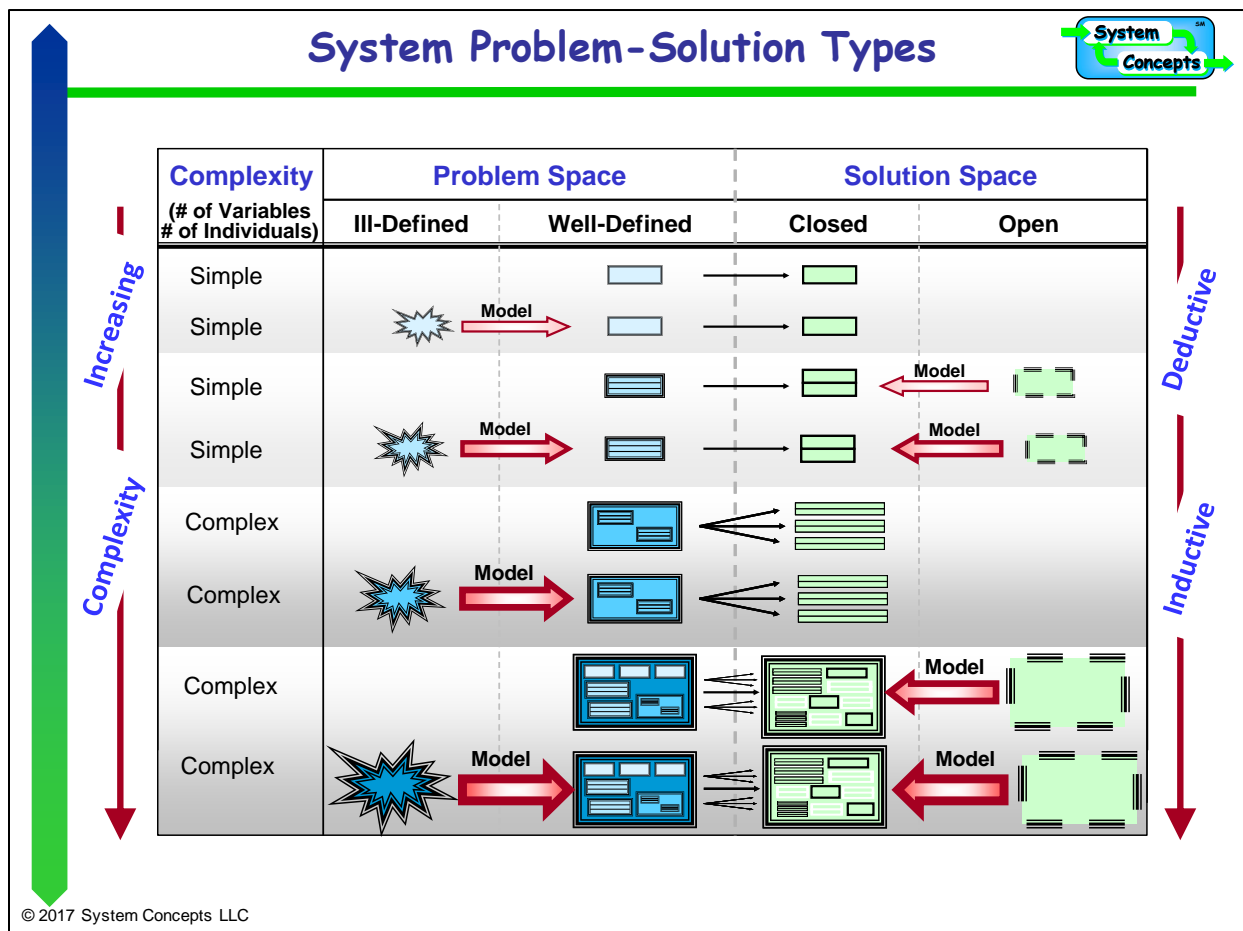


Figure 2. System Problem-Solution Types

B. Language Types

Structural modeling is used to identify and model the structure of a given problem space that is too complex for simple analysis and too organized for statistical analysis. In some cases, the given problem is simplified, using various techniques, in an attempt to move the problem into the simple analysis portion of the chart.

The simplification approach is a common mistake that rarely provides the bases for an acceptable solution. A more viable solution approach is to model the given problem structure to an adequate level of detail and then design solutions that address the modeled problem structure.

The first step in problem solving is the identification and evaluation of one or more problems to be solved. Problems exist in a common context and are related in various ways. Once the relevant problems are identified and placed in the proper context then a wide range of problem solving techniques and methods may be applied to seek viable solutions. As shown in Figure 2, many complex problems are not clearly defined or understood. The uncertainty associated with the problem definition is one aspect that tends to increase complexity. A second aspect associated with large-scale problems is the wide range and scope of the problem or problem set. Many problems are well beyond the capability of a single individual to understand at any relevant level of detail. Therefore, groups of individuals must be engaged to actively seek valid problem identification and definition. Once groups of individuals are needed in the process, issues associated with a common language, concepts and world views tend to add complexity to the structural modeling process. The three common language forms - prose, structured graphics and mathematics - are shown in Figure 3.

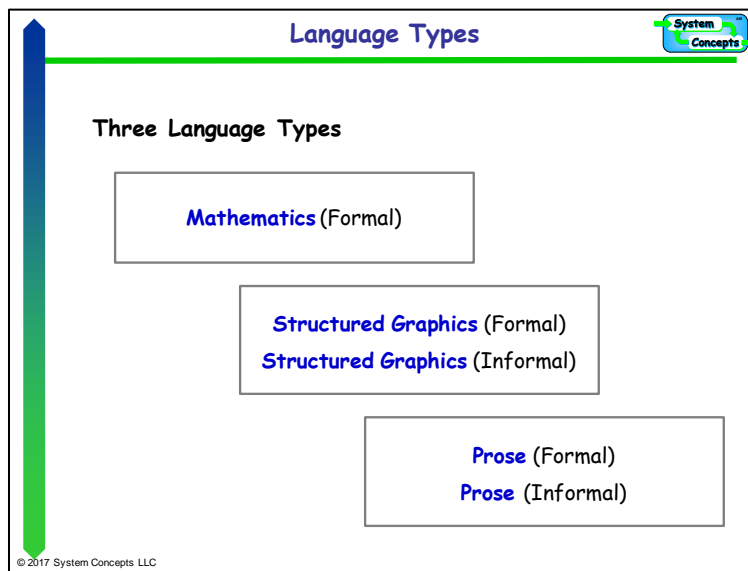


Figure 3. Language Types

C. Documenting Language Types

These three types of languages (prose, structured graphics, and mathematics) are each designed to convey different types of information. Each language form has its advantages and disadvantages when applied to large-scale problem identification and documentation. Human understanding and logical reasoning is constrained by a number of factors creating a situation where computer based logical reasoning is required to support the logical evaluation of systems that have more than a few objects. The Abstract Relation Type (ART) was developed as a standard framework for the recording and presentation of system information given in all three language forms, prose, structured graphics and mathematics [Simpson et.al., 2007]. See Figure 4, Abstract Relation Types. A key aspect of the system ART form is the ability to encode isomorphic relationship information in all three language types. This type of system information encoding provides a valuable semantic translation structure among the language types.

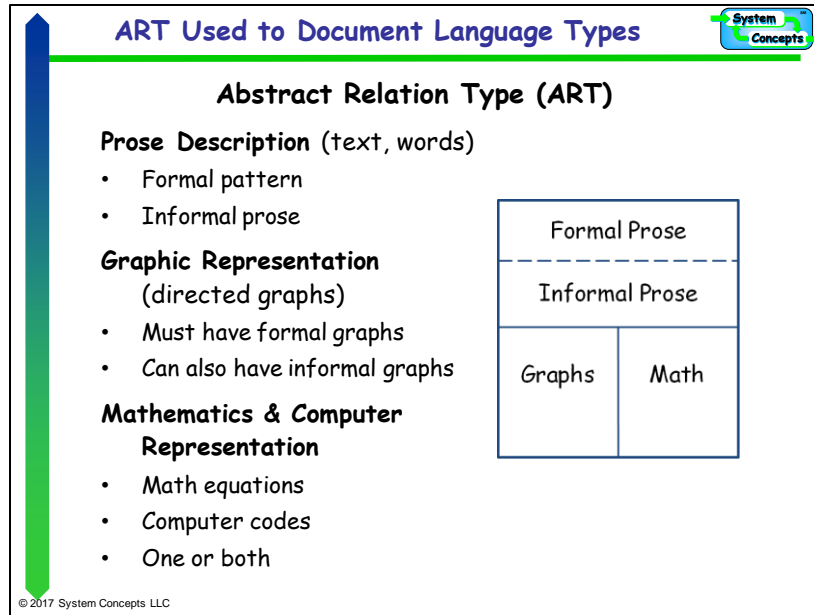


Figure 4. Abstract Relation Types

The system ART form is most valuable when the ART form is completed in a manner that supports an isomorphic language interpretation of the system relationship. However, an incomplete ART form also contains valuable information about a specific system structuring relationship. Using the fundamental concepts of logic and mathematical analysis, a set of system ART forms may be evaluated for existing data and information as well as missing data elements. A complete formal package of ART forms may be used as the basis of an automated system design and evaluation process. This type of automated process could then be aligned with other system representations like, UML and SysML.

D. Augmented Model-Exchange Isomorphism and Logical Properties

The Augmented Model Exchange Isomorphism (AMEI) was developed to record and encode a fundamental base set of isomorphic system relationship representations [Simpson and Simpson, Feb, 2015]. The AMEI presents logical system relationships in terms of the three language types aligned in an isomorphic manner. The model-exchange isomorphism (MEI) was developed by [Warfield, 1973] to highlight the advantages of an isomorphic transform between a binary matrix representation of a system structure and a structured graphical representation of the same system structure. One clear advantage of the MEI is the encapsulation and presentation of abstract mathematical representations in a structured graphical form. Individuals that do not understand the mathematical foundations and basis for the system structural relation can readily understand the system structure presented in the structured graphic representation. The MEI was transformed to the AMEI by adding the prose representation associated with a specific system structural relationship. The exploration and development of the AMEI highlighted a number of interesting factors including, the need for three logical property types and the role of the symmetric property in system structuring. Figure 5 provides an overview of the AMEI logical categories. Every system structuring relation must be aligned with each of the logical properties to produce a unique set consisting of a reflexivity property [reflexive, irreflexive or nonreflexive], a symmetry property [symmetric, asymmetric or nonsymmetric] and a transitivity property [transitive, intransitive, or nontransitive]. The combination of the AMEI and the system ART form create an environment where system structures, at various levels of abstraction, may be effectively encoded to support the development, application and exploration of general systems theory (GST.)

<h1>Logical Relation Properties</h1>		
<h2>Hi-Level Logical Characteristics of Three Dyadic Relations - v1.1</h2>		
<i>Reflexivity</i> <i>Involves one individual</i>	<i>Symmetry</i> <i>Involves two individuals</i>	<i>Transitivity</i> <i>Involves three (or more) individuals</i>
<p>Reflexive</p> <p>A relation, R, is reflexive iff any individual that enters into the relation bears R to itself.</p> <p><i>*Identical with; Divisible by</i></p>	<p>Symmetric</p> <p>If any individual bears the relation to a second individual, then the second bears it to the first.</p> <p><i>*Touching</i></p>	<p>Transitive</p> <p>If any individual bears this relation to a second and the second bears it to a third, then the first bears it to the third. <i>*Greater than; North of; Included in</i></p>
<p>Irreflexive</p> <p>A relation, R, is irreflexive iff no individual bears R to itself.</p> <p><i>*Stand next to; Father of</i></p>	<p>Asymmetric</p> <p>A relation, R, is asymmetrical iff, if any individual bears R to a second, then the second does not bear R to the first.</p> <p><i>*North of; Heavier than; Child of</i></p>	<p>Intransitive</p> <p>A relation, R, is intransitive iff, if any individual bears R to a second and the second bears R to a third, then the first does not bear R to the third. <i>*Father of; 2" taller than</i></p>
<p>Nonreflexive</p> <p>A relation which is neither reflexive nor irreflexive is nonreflexive.</p> <p><i>*Respecting; Killing</i></p>	<p>Nonsymmetric</p> <p>A relation which is neither symmetrical nor asymmetrical is nonsymmetric.</p> <p><i>*Likes; Seeing</i></p>	<p>Nontransitive</p> <p>A relation which is neither transitive nor intransitive is nontransitive.</p> <p><i>*Admiring; Fearing</i></p>
<p><i>*Examples</i></p>		

p. 14 of 30
Adapted from *Predicate Logic and Handbook of Discrete & Combinatorial Mathematics*

Figure 5. Logical Property Groups

III. GENERAL SYSTEMS THEORY

The wide range and volume of specific scientific and engineering theories presents an overwhelming amount of information and data. Individuals are struggling to effectively engage this vast data store that increases on a daily basis. The search for a general systems theory (GST) is an ongoing activity that attempts to engage this ever-increasing data store by identifying a general systems theory that applies to a wide range of specific domain theories and data. A GST would reduce a large number of specific domain theories to a single set of general systems theories that may be adjusted to fit a wide range of specific domain theories.

[Boulding, 1956] proposed the following definition of GST:

“General Systems Theory is a name which has come into use to describe a level of theoretical model-building which lies somewhere between the highly generalized constructions of pure mathematics and the specific theories of specialized disciplines.”

“It does not seek, of course, to establish a single, self-contained “general theory of practically everything” which will replace all the special theories of particular disciplines. Such a theory would

be almost without content, for we always pay for generality by sacrificing content, and all we can say about practically everything is almost nothing. Somewhere however between the specific that has no meaning and the general that has no content there must be, for each purpose and at each level of abstraction, an optimum degree of generality."

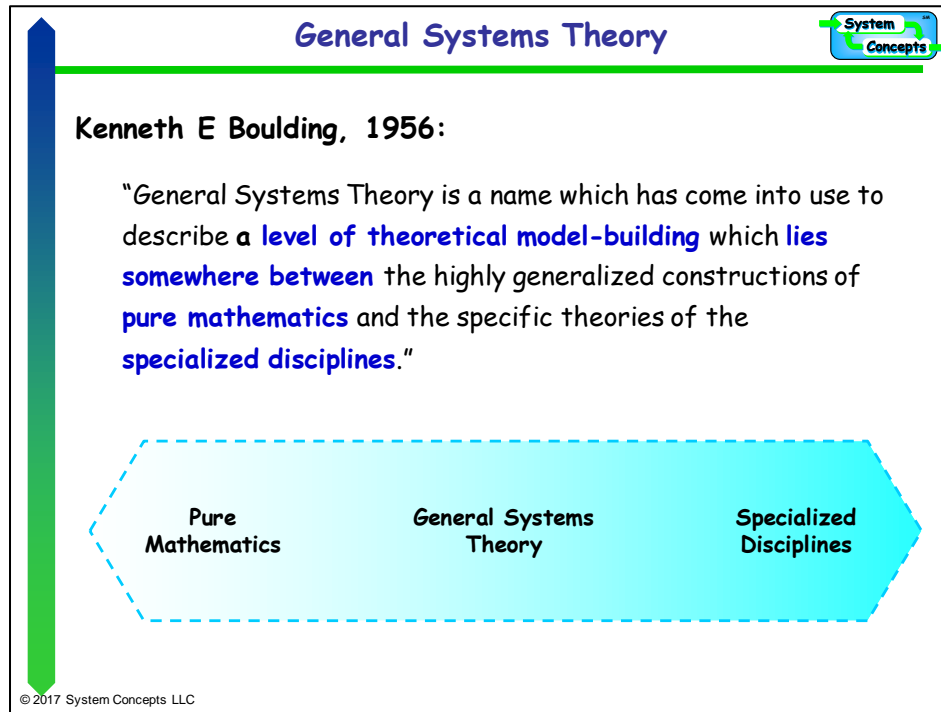
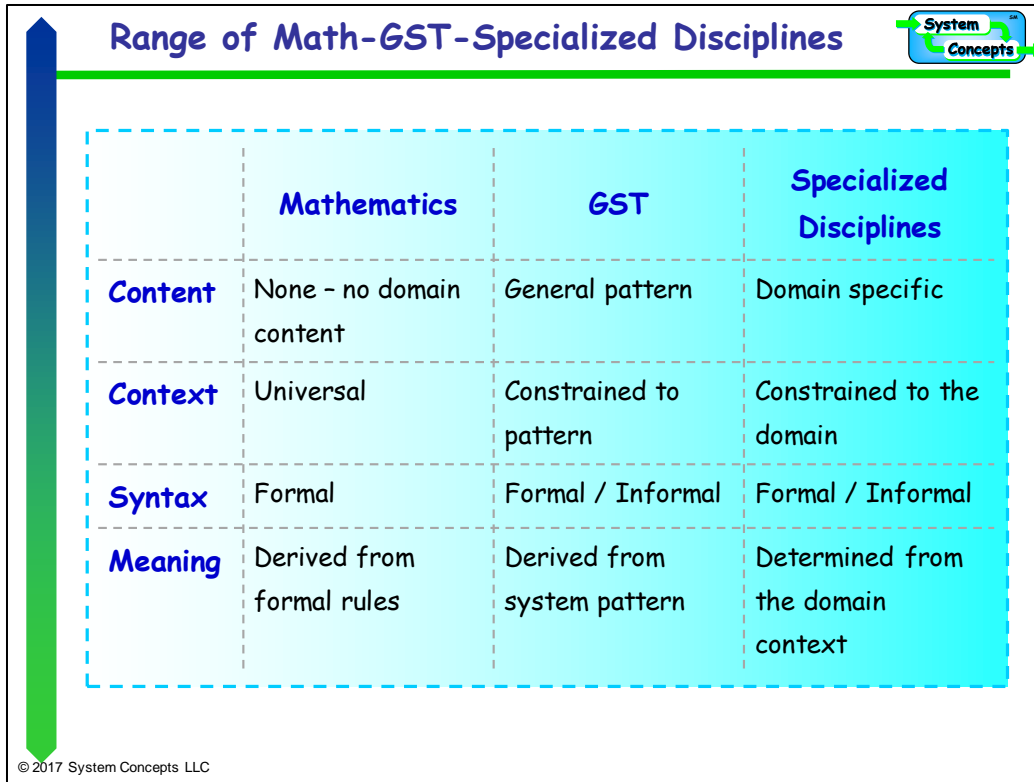


Figure 6. General System Theory

A. Range of General Systems Theory

The preceding material from Boulding presents some very interesting details for analysis by a system scientist or system engineer. A key detail is the claim that “between the specific that has no meaning and the general that has no content.” This specific claim brings into focus the concept of meaning and semantics. It is important to notice that different languages (mathematics and prose) are being evaluated and analyzed for meaning and content. Mathematics is a formal language, meaning is derived from formal rules, operations and syntax. Human prose is an informal language, meaning is determined from the current symbol (sign) set and a shared common context among the human group

Each language type has strong advantages. Mathematics, a formal language, may be used to process vast amounts of information that would be impossible to evaluate using an informal language like prose. Prose, an informal language, has the capability to add needed contextual detail to formal language constructs. Common names and domain-specific lexicons are examples where specific contextual information is added to a general theory to create a specific theory. Bolding’s description of GST provides a foundation upon which specific aspects and attributes of a GST may be evaluated. One of these aspects is the establishment of the boundary between mathematics and GST. What specific properties of a GST separate a GST from pure mathematics? Another one of these aspects is the establishment of the boundary between GST and a specific domain theory. What specific aspects of a unique domain theory makes it different from a GST? This is an area where significant progress may be made in the development and definition of GST (see Figure 7, GST Range).



The figure is a table titled "Range of Math-GST-Specialized Disciplines" with a logo for "System Concepts" in the top right corner. The table is organized into four rows and three columns. The columns are labeled "Mathematics", "GST", and "Specialized Disciplines". The rows are labeled "Content", "Context", "Syntax", and "Meaning". The table is enclosed in a dashed blue border. A vertical green bar is on the left side of the table, and a copyright notice "© 2017 System Concepts LLC" is at the bottom left.

	Mathematics	GST	Specialized Disciplines
Content	None - no domain content	General pattern	Domain specific
Context	Universal	Constrained to pattern	Constrained to the domain
Syntax	Formal	Formal / Informal	Formal / Informal
Meaning	Derived from formal rules	Derived from system pattern	Determined from the domain context

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Figure 7. GST Range

B. Structural Modeling and the General Systems Theory

The three general areas associated with the GST may be mapped to the three areas of structural modeling in a one-to-one fashion. Structural modeling has two original areas and a third area added in the last few years. The original two areas, developed by Warfield, are basic structural modeling (BSM) and interpretive structural modeling (ISM.) The third area, developed by Simpson, is structural integration modeling (SIM) [Simpson and Simpson, 2014]. The structural modeling areas are constrained to focus on the specific aspects that support the identification, understanding and documentation of system structure. Therefore, BSM only focuses on the mathematical operations that provide system structural insights and effective transforms. Similarly, ISM only focuses on the substantive, empirical information associated with complex systems and issues. The focus of SIM is the proper alignment and correspondence between BSM components and ISM components for any given domain situation. The SIM methods and processes are focused on providing added value by creating standard solution approaches, work flows and process patterns. See Figure 8 for an overview of the structural modeling areas and their function.

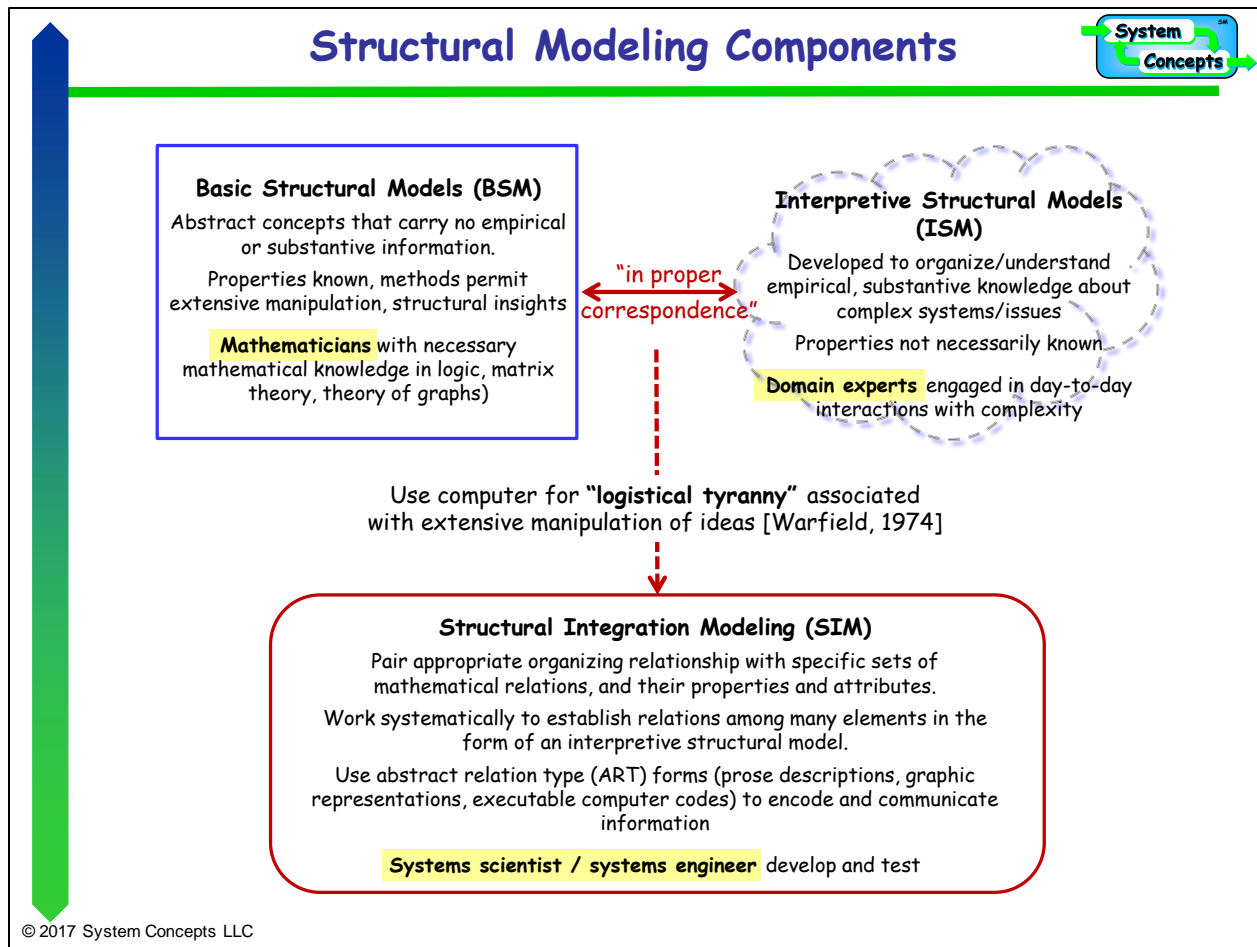


Figure 8. Structural Modeling Components

C. Structural Integration Modeling

The structural modeling activities associated with the SIM area create a rich, varied context filled with common structural modeling artifacts that support the development of a GST. In the SIM area, careful consideration must be given to the alignment of formal meaning from the BSM area and the substantive informal meaning associated with the ISM area. Some classical literature that addresses the sequencing of events in a project schedule and is also associated with ISM techniques proclaims that each event precedes itself, by definition! From an empirical, substantive data and information point of view, this claim seems unfounded. A closer look at the role of SIM reveals a different more powerful view of the actors in this analysis. From the BSM point of view, there is no available empirical, substantive information to inform the ordering of the events. Therefore, a partial ordering of the event set was used in the abstract evaluation. However, from an ISM point of view, there exists abundant substantive data and information that may be used to determine the system structure and event sequence. The empirical information is not available in the BSM area. When methods and techniques in the SIM area are used to properly align BSM and ISM approaches, this information and analysis disconnect must be addressed. A simple SIM transform may be used to address this specific situation and greatly reduce the complexity associated with the classical Design Structure Matrix analytical methods.

The mathematics of structure, in the BSM area, has strong structured graphics, matrix and logic components. Two foundational artifacts were created for use in the SIM area that supports the standard encoding and presentation of system relationship attributes. The first artifact is called an abstract relation type (ART) and was patterned after an abstract data type found in computer science [Simpson et.al., 2007]. The second artifact is called the augmented model-exchange isomorphism (AMEI) [Simpson and Simpson, Feb, 2015] which is based on the model-exchange isomorphism (MEI) developed by Warfield. These foundational artifacts support the analysis of system structuring relationship logical properties. Initial analysis of system structuring problems indicate that the symmetric property of a system structuring relationship controls the system basic structural form [Simpson and Simpson, Feb, 2015]. See Figure 9 for an overview of this general pattern

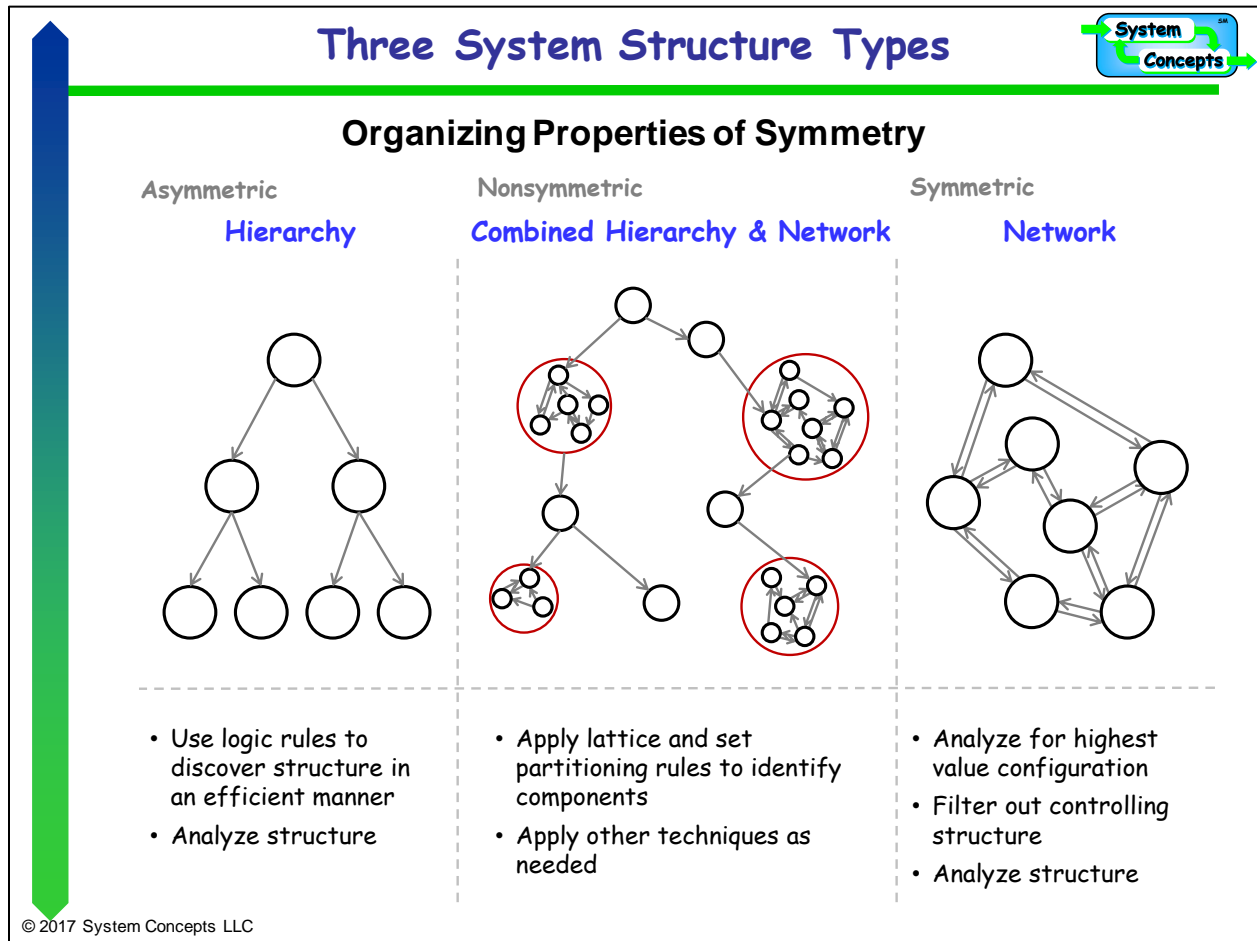


Figure 9. System Structural Types

IV. SYSTEM STRUCTURING RELATIONSHIPS

System structuring relationships are associated with a range of relationship attributes. Two relationship attributes that provide useful analytical support are named (1) global and (2) local [Simpson and Simpson, Dec, 2016]. A system relationship with a global attribute maps the relationship between and among objects using a mediating artifact. A system relationship with a local attribute maps the relationship directly between and among objects without a mediating artifact.

Systems that are structured using relationships with global attributes can be further categorized by identifying the type of symmetric property (asymmetric or symmetric) and the type of global relationship (objective or subjective.) Four system structuring relationships are addressed next. These four relationships are:

1. 'is heavier than': objective and asymmetric
2. 'is north of': objective and asymmetric
3. 'in the same category as': subjective and symmetric
4. 'is of higher priority than': subjective and asymmetric

Structural threads are artifacts created on structural graphs when two or more objects are connected using a relationship link. Some types of structural relationships produce a single structural thread on the system structural graph. Other types of relationships produce more than one structural thread on the system structural graph. In a few cases it is possible for a malformed or degenerative structural thread to be created. A malformed or degenerative structural thread is composed of a single object. Given a single object there is no other object to support a relationship connection.

A. 'Is Heavier Than' Relationship

The 'is heavier than' relationship is an example of a global relationship that uses the global gravity field as a mediating artifact as well as the basis for evaluating a linear system structure comprised of objects ordered by weight [Simpson, et.al., March, 2017]. As shown in Figure 10, the 'is heavier than' relationship generates a single structural thread for the complete system. All items have a different distinct weight, one item per weight class.

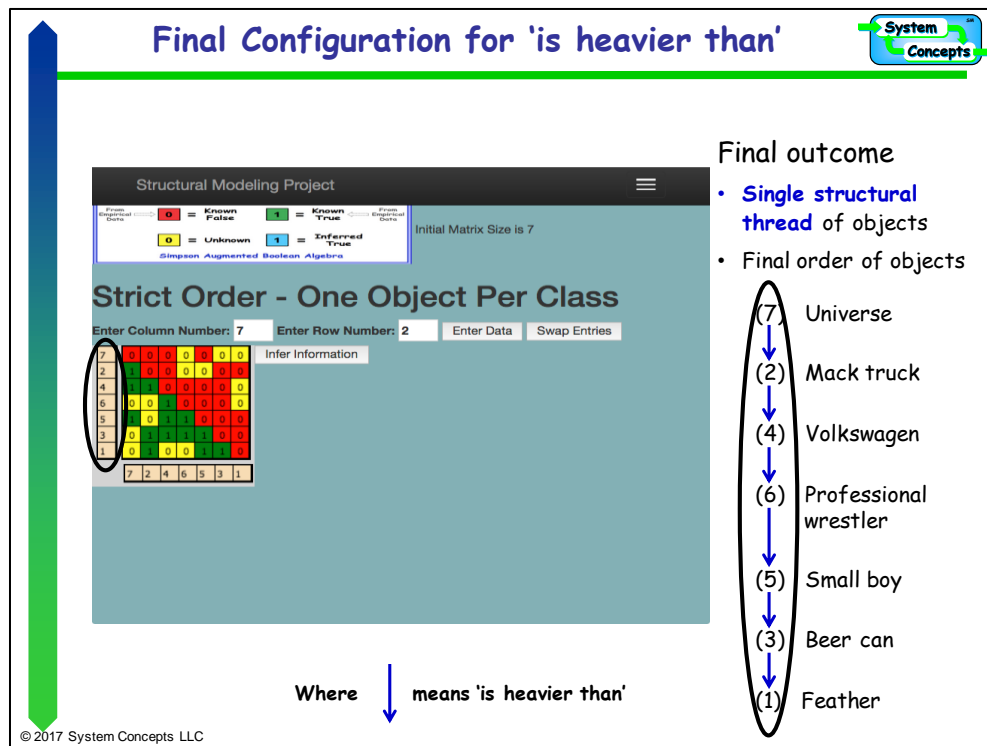


Figure 10. Single Structural Thread

B. 'Is North Of' Relationship

The 'is north of' relationship creates a single structural thread through the set of latitude location classes. If two cities are at the same latitude, then they are placed in the same location class. Like the previous 'is heavier than' example, the 'is north of' relationship is an objective, global property. Unlike the 'is heavier than example', the 'is north of' relationship has more than one object per class. This general approach of using only strict ordering is very effective at removing the semantic and interpretation issues associated with the use of partial ordering. From an operational point of view, if two objects are equivalent they are placed in an equivalence class and the equivalence class is used as the object to be ordered. The ability to gather empirical information, for any given SIM task, from the ISM structural modeling component facilitates the use of strict ordering. Figure 11 provides an overview of the 'is north of' system structure.

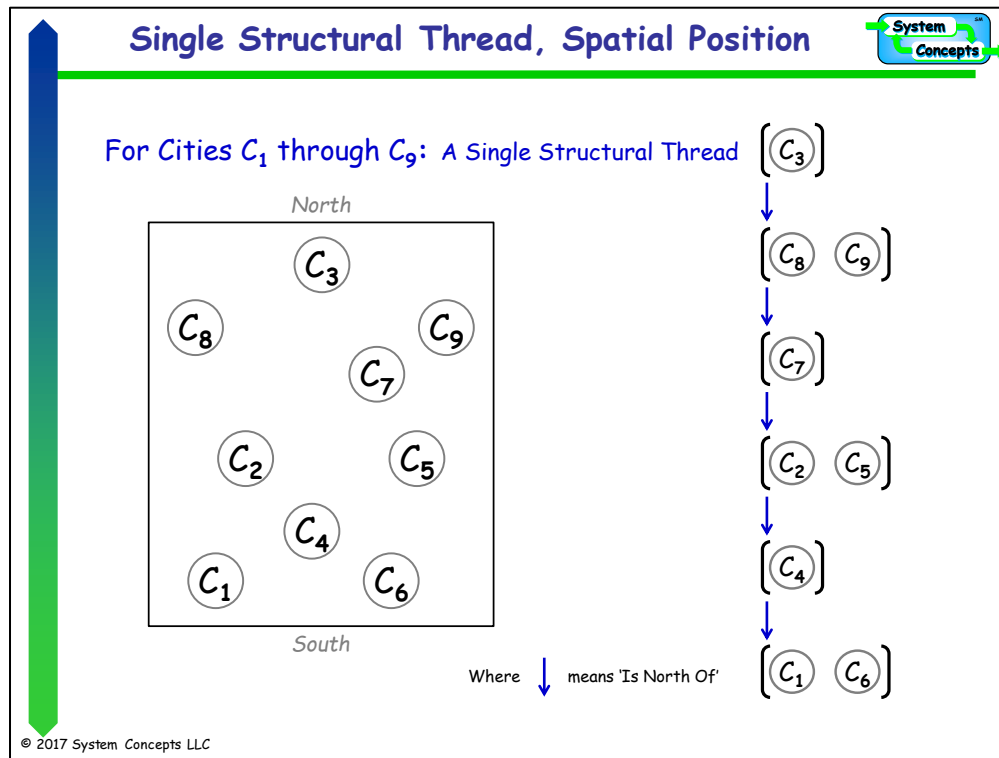


Figure 11. 'Is North of' Example

C. 'Is in the Same Category as' Relationship

The 'is heavier than' and 'is north of' system structuring relationships are both global, objective and asymmetric. The 'is in the same category as' system structuring relationship is global, subjective and symmetric. The subjective nature of the global artifact indicates the need for a focused activity that creates a common understanding of the subject categories and their controlling identifying attributes. Given the same object set, two different groups of individuals may well generate different categories and object assignments for the category groups. The symmetric attribute is interesting and it is required to support the equivalence among objects in the same category. Under these conditions each category generates a single structural thread. There are no categories that are empty, each category must have at least one object. A category with only one object generates a degenerative structural thread and is an indication that the general

set of categories should be reevaluated to eliminate categories with only one object. See Figure 12 for an example of a system structure with three structural threads

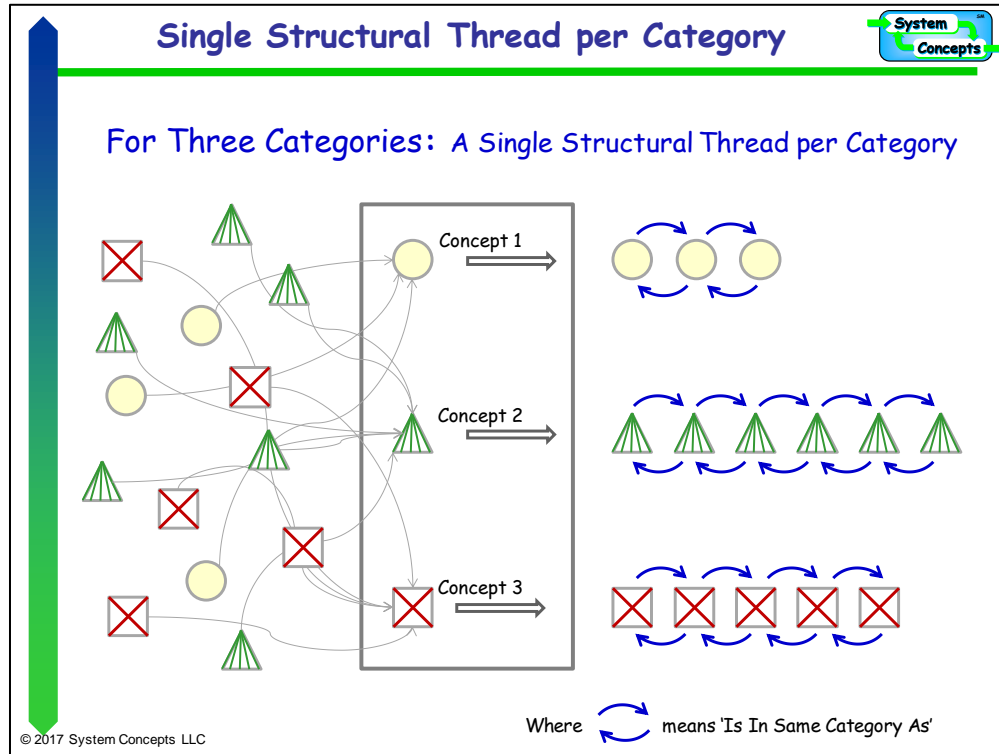


Figure 12. Three Structural Threads

D. 'Is of Higher Priority Than' Relationship

Similar to the 'is in same category as' relationship, the 'is of higher priority than' relationship is global and subjective. Unlike the 'is in same category as' relationship, the 'is of higher priority than' relationship is asymmetric with only one global structural thread. While this single priority thread can contain more than one objective at any given level in the priority scale, the structure is more effective if there is only one objective per level.

The main function of an objective prioritization activity is the identification of a clear set of priorities. A structure that has only one objective per level is the best way to clearly present a set of priorities. The subjective attribute associated with this global structuring relationship indicates a need for special attention to the development of a common understanding among the individuals engaged in the priority setting activity. Unlike the force of gravity that is used as a global mediating artifact in the 'is heavier than' relationship, the global mediating artifact for 'is of higher priority than' is a concept that needs to be discussed and evaluated by the group performing the prioritization. Given two different groups of individuals, each group may generate very different priority structures given the same object set for each group. In some cases, the relationship between the objective set and the priority structure may also need additional evaluation. Figure 13 presents an overview of a priority system structure.

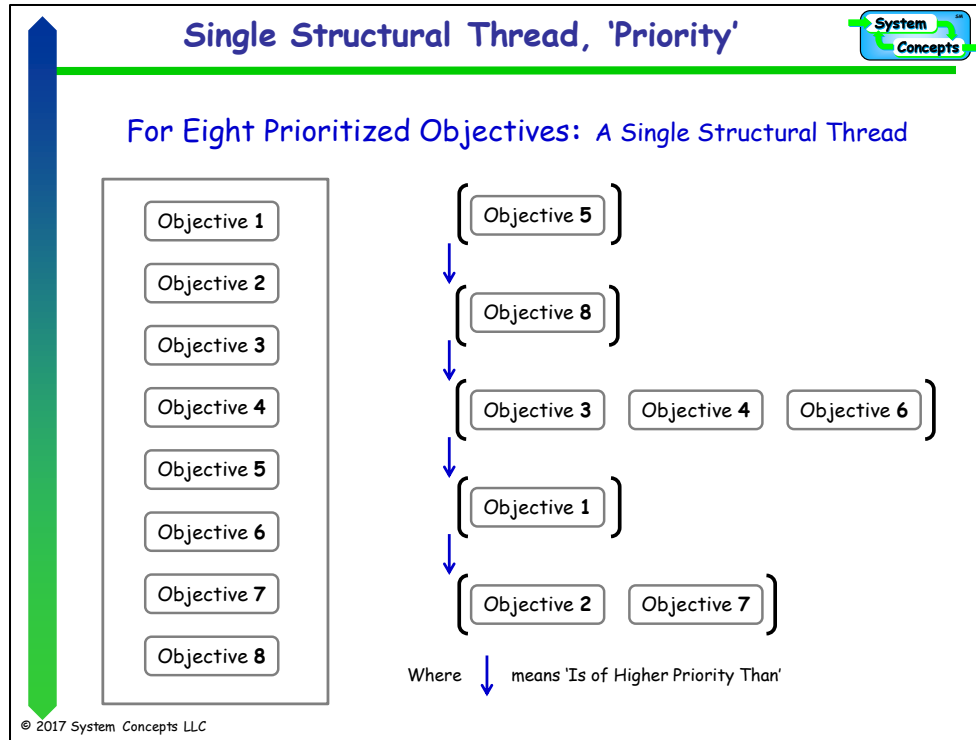


Figure 13. Priority Structure

V. SUMMARY AND CONCLUSIONS

Structural modeling is a general approach to the solution of complex problems. Classical structural modeling techniques are now enhanced and coordinated with a new component, structural integration modeling (SIM.) The refined structural modeling techniques have a basic one-to-one correspondence with techniques associated with general systems theory (GST.) This relationship can be used to inform and further develop both structural modeling and GST.

The abstract relation type (ART) form and the augmented model-exchange isomorphism (AMEI) are two system analysis artifacts that support more detailed, refined analyses in structural modeling. The added value stems from the common information format and methods encoded in the ART form and the AMEI transforms. Structural modeling contains a critical software component that is used to support the evaluation of a given current system structural state. Logical analysis of the system state, by the software, minimizes the resources necessary to discover the unknown or poorly defined system structure. The ART form and the AMEI transforms provide necessary inputs to the development, production and operation of structural modeling software.

Common characteristics associated with specific system types are used to identify specific software functions as well as the required range of software support functions. The logical property groups presented in the AMEI are key discriminators that are used for the selection of general solution approaches. In addition to system relationship logical property groups, system relationship attributes of global, local, objective, and subjective play important roles in forming an effective evaluation process. Continued research into open source structural modeling software provides a forum for the identification, discussion, evaluation and analysis of a wide range of system relationship attributes.

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