System Analysis and Identification: Objects, Relations and Clusters

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Abstract:

The use of Abstract Relation Types (ART) in the analysis of system structure and system component clustering is the primary focus of this paper. Two basic system definitions are presented along with two, object-clustering definitions which were obtained from a literature search. Systems structuring mathematical properties, used in systems analysis, are outlined and discussed. The ART analysis approach is applied to classical N-Squared Charts and Design Structure Matrices (DSM), with specific emphasis on clustering methods, types and meaning. The primary structuring relationship associated with N-Squared ART and DSM ART are evaluated and discussed. Multiple DSM ART solution approaches and techniques are detailed.

Key Words:

Systems structuring mathematical properties

Introduction:

For the purposes of this paper, a system is defined in two, complementary ways: a construction-rule definition and a function-rule definition. The construction-rule definition of a system [Simpson and Simpson, 2003] is "A system is a relationship mapped over a set of objects." The function-rule definition for a system [Heylighen, 1994] is "A system is a constraint on variation." The construction-rule system definition is the foundation of a number of classical system engineering graphical analysis and representation techniques including N-Squared Charts, Automated N-Squared Charts and Design Structure Matrices. The function-rule definition explores specific system configurations and determines whether these configurations are feasible, optimal and/or applicable to a specific system and deployment context. The construction-rule definition is applied to both the system objects and the system organizing relationship in a concurrent fashion. The constraints associated with the function-rule definition may be applied to the objects only, the relationship only, or to a combination of both the objects and the relationship. The function-rule definition is mainly applied to the analysis of the system object sequence represented on the matrix diagonal of the examples presented in this paper.

Clustering, or the activity of creating clusters, is one key focus of this paper. Clusters are defined in two complementary ways: object-based and space-based. The first definition of clustering [Tryon and Bailey, 1970] for cluster analysis***:*** "objectively group together entities on the basis of their similarities and differences." This object-based definition requires that the objects in the system be identified and evaluated using a common set of factors. The activity of object-based clustering identifies objects of interest, analyzes and groups the identified objects' with similar factors, and generates a space populated with the object clusters. The second definition of a cluster [Warfield and Hill, 1972] is: "a cluster is a set of entities that lie in a closed connected subspace of some space." This space-based definition of clustering starts with the contextual space, locates a subspace of that contextual space and then identifies the objects in the subspace as a cluster. The activity of space-based clustering requires three steps: (1) identify the controlling contextual space, (2) identify a subspace within the controlling contextual space, and (3) enumerate the entities (or objects) that are located in the subspace. Figure 1 captures a notional view of the two, complementary definitions of cluster.

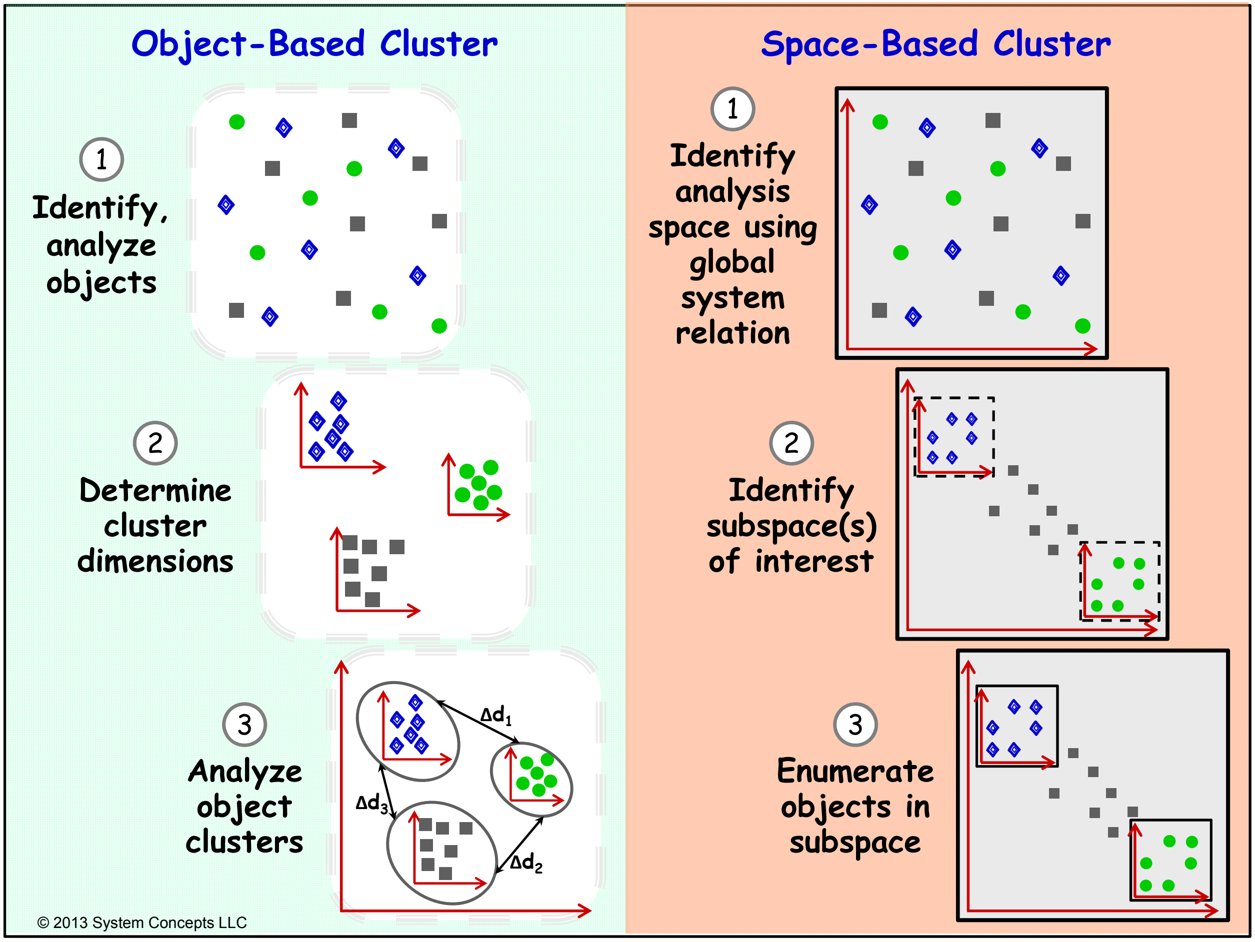


Figure 1. Two Main Types of Clustering Processes

These complementary definitions and views of a cluster, and the activity of clustering, provide the foundation needed to directly connect a system and a cluster using the definition of a relation provided by [Weiner, 1914]. Weiner's definition of relation reduced the theory of relations to the theory of classes, and added clarity and rigor to the concept of a relation. [Warfield and Christakis, 1987] present the Weiner relation definition as: "a relation is a subspace of a space. Substituting this relation definition into the construction rule definition of a system, a new definition becomes: 'A system is a subspace of a space that is populated with the system objects.' This new definition of a system appears strongly similar, if not equivalent, to the Warfield/Hill definition of a cluster. A high-level review of the literature reveals a strong congruence between the formal idea of a cluster, and that of a system.

Key cluster analysis literature is reviewed, encapsulated and initially presented in the 'Historical Context' section. Clustering techniques developed by the authors are then applied to a classical N Squared Chart example to support the discussion of the general techniques and processes. In the next section of the paper, clustering techniques are applied to a set of Design Structure Matrix (DSM) examples. A small number of differing DSM examples are presented from the published literature. The apparent differences among historical DSM publications are discussed in terms of the structural modeling work of Warfield, the authors' clustering techniques, and current DSM clustering techniques.

An integrated system and clustering structure is then presented by the authors. This integrated structure has a global organizing system relation that creates the structure for the global system space. The global system space contains the subsystem cluster space. This dual level of cluster identification first constructs a global system space, using a known global system relation; second, it discovers the local cluster relation using appropriate analytical techniques. The Abstract Relation Type (ART) is used to organize and present the prose, graphic and mathematical structure associated with the ART Automated N Squared Charts and the ART Design Structure Matrix activities. The system ART form creates a set of equivalent prose, graphic and mathematical constructs that describe and detail the application and use of the specific system relation or set of relations.

Historical Context:

Cluster analysis is a structured process using scientific methods focused on the discovery of general properties of objects, and the general types into which objects may be categorized or classed.

- The activity of identifying general properties of objects is termed "V-analysis" or variable analysis. The objective of V-analysis is the identification of the degree of similarity among the variables that are used to identify and describe the object properties. Different types of V-analysis can be used to identify the dimensions upon which the objects will be clustered and evaluated.

- The activity of identifying the general types into which objects may be categorized or classed is termed "O-analysis" or object analysis. The objective of O-analysis is the construction of a formal, planned, scientific classification. Object analysis activity can be traced back before the time of Linnaeus in 1753 to the work of Aristotle. Object analysis is not just the mere placement of objects in predefined classes but also includes the identification and naming of the object class types.

[Tryon, 1939] produced *Cluster Analysis* – a book written to simply and clarify professional practices of determining psychological differences among groups of people. His statistical methods and techniques were developed as "desk instructions" for manual calculation in the 1930's, 40's, and 50's. In the 1950's as computers evolved, these basic techniques were translated to mainframe computer programs. A new version of his book was published that detailed the history and development of the practice of cluster analysis (Tryon and Bailey, 1970].

[Steward, 1962] published "On an Approach to Techniques for the Analysis of the Structure of Large Systems of Equations," a paper which detailed processes and procedures for the efficient application of computer resources to the solutions of large numbers of equations. The constructive techniques described in this paper identify groups of equations with common loops that are contained within partitions. No variable data values flow across the partition boundaries. The solutions of these equation groups obey a precedence relationship. This technique creates clusters of equations that have common variable exchange loops.

[Steward, 1965] published "Partitioning and Tearing Systems of Equations", which expanded on the work in his 1962 paper. This 1965 paper presents a detailed algorithm used to group common equations into blocks (or clusters).

[Warfield and Hill, 1972] introduced three dimensions of a 'Systems Engineering Framework' upon which all activities associated with systems engineering could be mapped and clustered.

Dimensions of the systems engineering tool space, and the dimensions of ‘transportation method clusters,’ were also included in this monograph.

In the monograph, “An Assault on Complexity”, [Warfield, 1973] introduced the ‘Parson Pattern Variables’ which assist in the construction of a dimensional range of solutions to societal problems. This dimensional range provides the variable space upon which candidate solution approaches may be clustered and evaluated. Mathematical techniques for discovering and evaluating feasible plans and objects are also presented, along with the previously mentioned definition of a cluster.

In the monograph - “Structuring Complex Systems, [Warfield, 1974] presents a set of mathematical techniques that are used to develop a system structure that is more readily communicated and understood by the engineering and management teams. These techniques include partitioning of matrix spaces, and the identification of partially-ordered system groups and/or clusters.

Another early work in cluster analysis is Clustering Algorithms [Hartigan, 1975]. This work defines clustering as the grouping of similar objects, and a cluster as a set of similar objects. Hartigan's work appears to mostly focus on the object analysis types of activities.

Societal Systems [Warfield, 1976] further developed methods and procedures of structural modeling, including detailed mathematical information on system structuring with binary matrices, as well as the partitioning of these systems into different spaces (clusters) depending on the objective of the analysis.

[Warfield and Christakis, 1986] detail a partitioning approach that has four levels: Level One: Target Level; Level Two: Cluster Level; Level Three: Dimension Level; and Level Four: Option Level. Level Three is similar to V-analysis activities in cluster analysis, with a focus on the dimensions (or variables) of the cluster space, while Level Two is similar to O-analysis focusing on the groups of objects to be assessed.

This paper also further developed the concepts and techniques associated with dimensionality and clusters as they apply to the structuring of complex systems.

The application of clustering techniques in science and systems science, as documented above and illustrated in Figure 1, shows two distinct and well-developed types of clustering practice. In the next section, abstract relation types (ART) identify components in the space-based clustering process: the system space, a subsystem space, and objects and items therein.

Abstract Relation Type (ART) Approach

N Squared Charts (N2C) and Automated N Squared Charts (AN2C) are standard methods of systems analysis used in the system engineering community, as well as being a specific type of interpretive structural model. The abstract relation type (ART) is used as the primary analytical technique by the authors for these two analytical methods. The authors named the analytical technique ‘abstract relation type’ to highlight and emphasize the primary role of the contextual relation in the application of an interpretive structural model. The contextual relation is an interpretation of a natural language statement of an organizing system relationship that is based on empirical data. The contextual relation is a fundamental component of structural modeling, or structuring of complex systems, which has two generic types: (1) basic structural models and (2) interpretive structural models. The mathematical properties associated with basic structural models are integrated, emphasized, and displayed using the ART technique as shown in Figure 2 and in Figure 3.

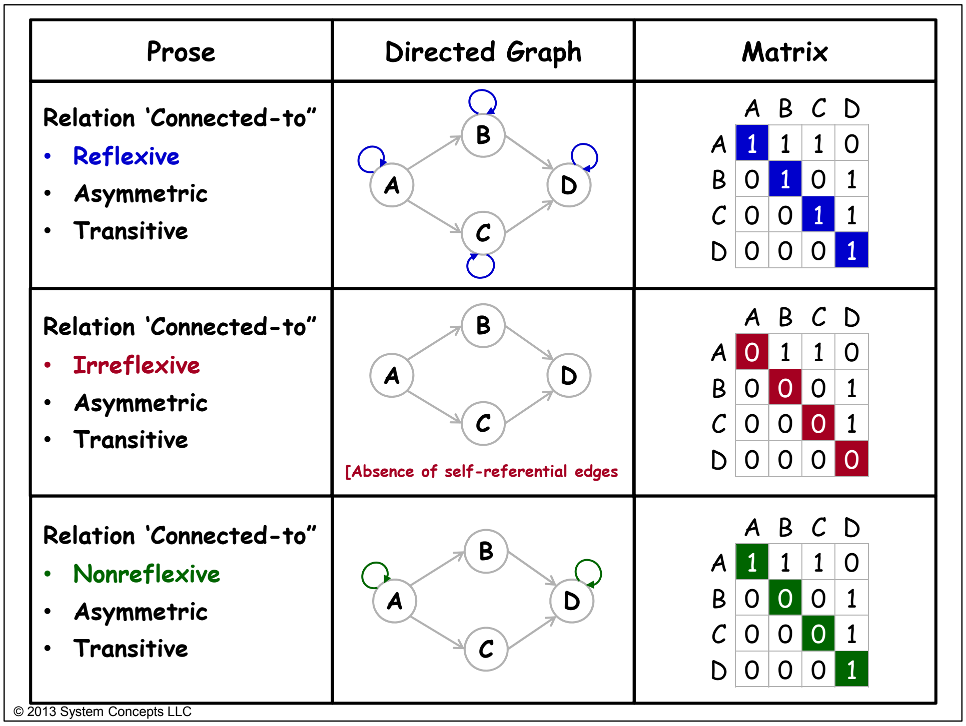


Figure 2. ART Equivalent Forms for **Asymmetric** Prose Properties

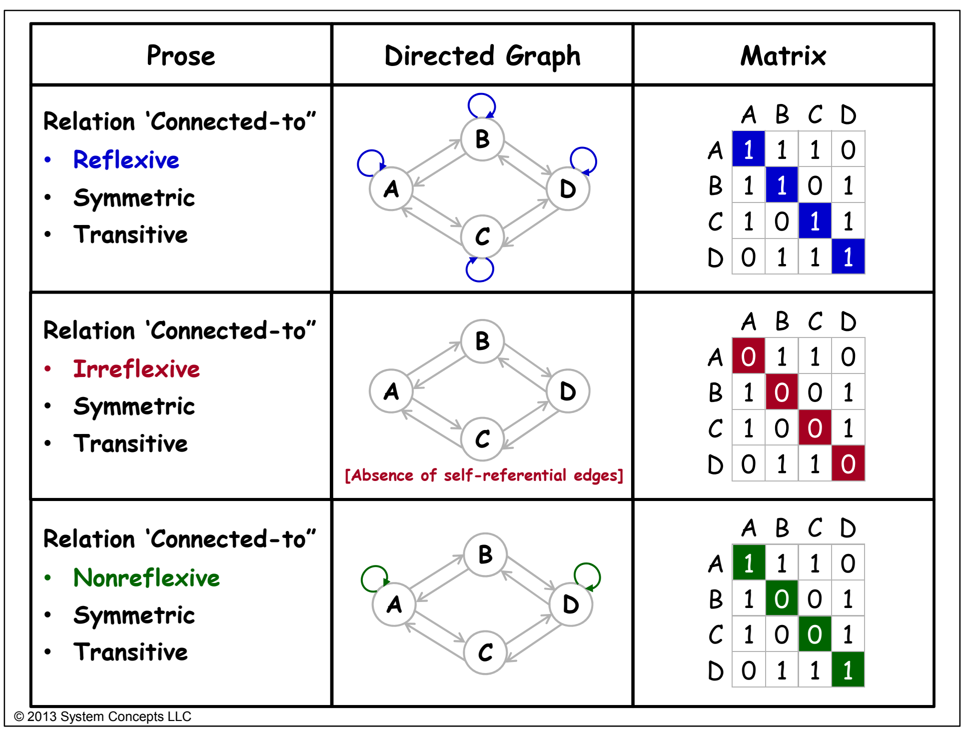


Figure 3. ART Equivalent Forms for **Symmetric** Prose Properties

[Warfield, 1974:i] defines interpretive structural models as:

> *"…those developed to help organize and understand empirical, substantive knowledge about complex systems or issues. Intent structures, DELTA charts, and decision trees, …, are examples of interpretive structural models. Other examples include interaction graphs, PERT diagrams, signal-flow graphs, organization charts, relevance trees, state diagrams, and preference chart."*

The ART approach in this paper is predominantly concerned with consideration and development of basic structural models that are used to augment the N-Squared Chart and Automated N-Squared Chart interpretive structural model types. As seen in comparing the contents of Figure 2 and Figure 3, the natural-language relationship 'connected-to' can be represented in a number of different configurations, depending on the properties of the relationship. Specifically addressing the properties of the relationship provides the capability to uniquely identify the proper mathematical and graphic form.

Automated N Squared Chart (AN2C) Example

The following description of Derek Hitchins' AN2C example (based on his graphs and text), demonstrates how the ART AN2C analysis is applied [Hitchins, 2003]. The directed graphs and matrix representations for this problem are analyzed to determine the mathematical properties of the connected-to binary relation. That is, the mathematical properties of reflexivity, symmetry, and transitivity are evaluated. See Figure 4 for the adjusted figure.

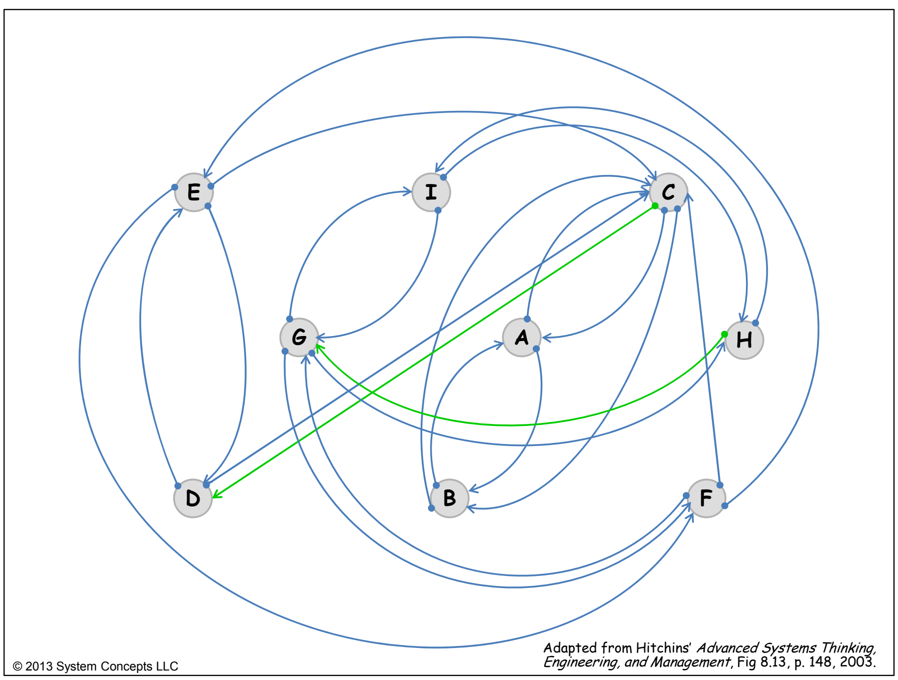


Figure 4. Hitchins' Directed Graph Representation

The *analysis for reflexivity* shows:

- The system digraph has no self-referential loops.
- The system matrix has no numerical values on the matrix diagonal.

Therefore, the connected-to contextual relation was assigned an irreflexive mathematical property.

The *analysis for symmetry* of the relation is also based on Hitchins' presented material:

- In the directed graph, it is clear that the edge arrows are single headed.
- Further, the definition of a basic N-Squared Chart requires one-way, directed connections. Two nodes may have a cyclical flow between each other, but two asymmetric connections are required.

Based on this information, the connected-to contextual relation was assigned an asymmetric mathematical property.

The *analysis for transitivity* of a relation is primarily derived from the common use of the natural language term, 'connected-to.'

- As shown in Figure 2, if node A is connected-to node B, and node B is connected-to node C, then node A is connected-to node C, which demonstrates the transitive nature of the connect-to relation.

Due to the directional and asymmetric nature of the connections, there may be cases where the transitive property should be further evaluated. In this case, the connected-to contextual relation was assigned a transitive mathematical property.

Equivalent forms of prose, graphics and mathematics for system representation required by the ART system analysis technique, provide multiple, reinforcing descriptions of the system organizing relation. These descriptions create an in-depth set of system information. Figure 5 demonstrates the application of the ART technique to the Hitchins AN2C example with both the disordered and the ordered system configurations. The ART connected-to relation is a global system relation that applies in any system configuration. It clearly defines the global system space structural relationship. The system may be ordered in a manner that creates the connection groups shown in Figure 5. These connection clusters define several subsystems in the system space. The objects in the subsystem space may be evaluated to determine common characteristics and/or other factors of interest.

Disordered System Configuration

E	0	1	0	0	0	1	0	1
0	I	0	1	0	1	0	0	0
0	0	C	0	1	0	1	1	0
0	1	0	G	0	1	0	0	1
0	0	1	0	A	0	0	1	0
0	1	0	1	0	H	0	0	0
1	0	1	0	0	0	D	0	0
0	0	1	0	1	0	0	B	0
1	0	1	1	0	0	0	0	F

Ordered System Configuration

A	1	1	0	0	0	0	0	0
1	B	1	0	0	0	0	0	0
1	1	C	1	0	0	0	0	0
0	0	1	D	1	0	0	0	0
0	0	1	1	E	1	0	0	0
0	0	1	0	1	F	1	0	0
0	0	0	0	0	1	G	1	1
0	0	0	0	0	0	1	H	1
0	0	0	0	0	0	1	1	I

Figure 5. ART AN2C System and Subsystem Spaces

The authors developed evolutionary computational techniques that are based on the *global* system relation, and depend on the mathematical properties of the contextual relation. In essence, the ART evolutionary technique is executed at the system level, not the component level, to provide a type of system computation that is focused on the complete system. The consideration of a relation's properties has also been used to address complexity reduction in fuzzy rules sets [Simpson and Dagli, 2008]. The two dimensional nature of a matrix representation is very similar to the two dimensional nature of a fuzzy number. When a matrix is used as part of the ART process, it may be referred to as a 'system number.' The key value structural configuration and value distributions are captured in these matrices much like a fuzzy number encodes the value and configuration associated with any specific empirical situation. The ART technique presented in this paper incorporates system structuring methods from basic structural models and interpretive structural models developed by Warfield. A key advantage of this ART approach is that many of Warfield's foundational concepts can be applied by a single individual to a situation as that individual understands it - without a large organizational commitment to processes and procedures.

In the AN2C example presented here, a single individual can identify the system structure. The system exists; it is operating; but it is not well documented with clearly defined and communicated boundaries. This type of situation is quite common, and will become more prevalent in the future as more and more systems are interconnected in an ad-hoc fashion. The system components are defined to be physical objects that are connected by physical artifacts. A wide range of system types are covered in this class of problem. One goal of the ART development is the application of system structural modeling techniques, developed by Warfield, to a much wider set of problem types using a variety of modern computing components.

In the next section, design structure matrix examples are evaluated and analyzed in terms of the ART analysis approach.

Design Structure Matrix Examples

Design structure matrices (DSM) were developed by Donald V. Steward to address the computational and cognitive complexity associated with the solution of very large systems of algebraic equations . The concept of information flow and equation system partitioning were created as conceptual tools used to reduce the complexity associated with the solution of systems of equations [Steward, 1961]. An algorithm for partitioning a large system of equations was developed which focused on the identification of "predecessors evaluations" that must be addressed before the current equation, or equation set, can be solved. The DSM techniques use a structural matrix that reflect a mapping of the set of equations onto the set of variables, as well as a structural matrix that reflects the mapping of the set of equations onto itself. These techniques are focused on the 'individual variable' and 'equation level' interactions. The precedence relationship among the clusters of equations is determined using a well-defined process that eliminates the information flow chains between the equations in a cluster, and leaves only the information flow chains between the clusters of equations [Steward, 1965].

There are a number of tightly integrated structuring concepts associated with the DSM equation evaluation processes. One concept is the identification and assignment of dependent variables and independent variables. In DSM evaluation processes, each equation is assigned one unique dependent variable, with all other variables in the equation being independent variables. The dependent variables are placed on the matrix diagonal, and used in the analysis of the system structure. The independent variables are placed, as applicable, in matrix cells that are not on the diagonal. Another concept is the impact evaluation of a specific variable in a set of equations. If a variable has little impact on the system solution then the specific value of the variable does not have to be well defined. If a variable has a large impact on the system solution, then the variable should be well defined and properly placed in the system equation set. The precedence relationship among groups of clustered equations in a precedence matrix is also among these tightly integrated system structuring concepts associated with the DSM equation evaluation processes.

DSM development expanded to include other types of systems including scheduling and project task analysis along with cause and effect structural relations. The 'Design of an Electric Car' example from *Systems Analysis and Management: Structure, Strategy, and Design* will be used as an example of the application of the ART DSM technique [Steward, 1981]. This specific DSM example is approached using three different, distinct ART DSM approaches. The first ART DSM approach was developed, executed and results reported in [Simpson and Simpson, 2009], and was based on the similarities between N Squared Chart system evaluation and DSM system evaluation. These similarities include:

- Square matrix system structure representations
- Semantic meaning associated with the upper matrix triangular area
- Semantic meaning associated with the lower matrix triangular area
- All system structural information encoded on one square matrix
- Asymmetric connections between nodes.

While the N Squared Chart has clear system component connection semantics that allow no empty rows in a N Squared Chart, the connection semantics associated with the DSM approach is defined to allow the construction of valid matrix configurations that have empty rows. These empty rows indicate a set of components that are not subject to the global system organizing relationship that is the basis of the ART method.

As shown in Figure 6, the basic ART has three fundamental spaces: the marking space, the value space, and the outcome space. Each of these three spaces is populated by one or more matrix representations or system numbers.

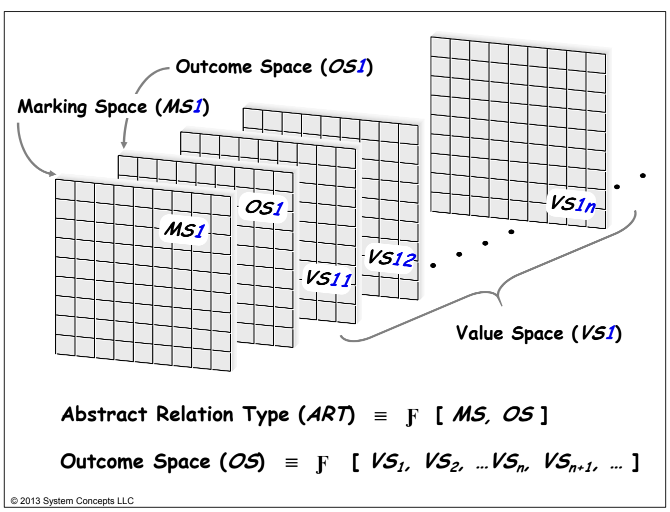


Figure 6. Abstract Relation Type

The marking space is populated by a matrix set that indicates the system structure. The value space is populated by a matrix set that indicates the system-of-interest value set. The outcome space reflects the values generated by mapping the system relation over the system structure. In the first ART DSM evaluation of the electric car problem, the marking space was populated with system connection marks only. The value space was designed to reflect the asymmetric, irreflexive and transitive nature of the DSM analysis approach. The DSM approach places a higher value on the lower triangular system structural marks in this example.

Because of the strong similarities between N Squared Charts and DSM, the “connected-to” relation was used in the first ART DSM analysis to evaluate the acceptability of using this natural language relation in the ART DSM approach. The initial marking space matrix for this example is shown on the left hand side of Figure 7 with the presence of a system structural connection given by a 1 in a matrix cell and the absence of a system structural connection given by a 0. The resultant marking space matrix for this example is given in Figure 7 on the right hand side of the figure. This first basic evaluation of the electric car problem using the ART DSM approach provided the same resultant system structure as the published classical DSM analysis. As shown on the right hand side of Figure 7, the resultant matrix has five empty rows at the top of the matrix which indicates the absence of any organizing system relation among these five entries. This outcome motivated the authors to continue with the development of more refined ART DSM analytical methods.

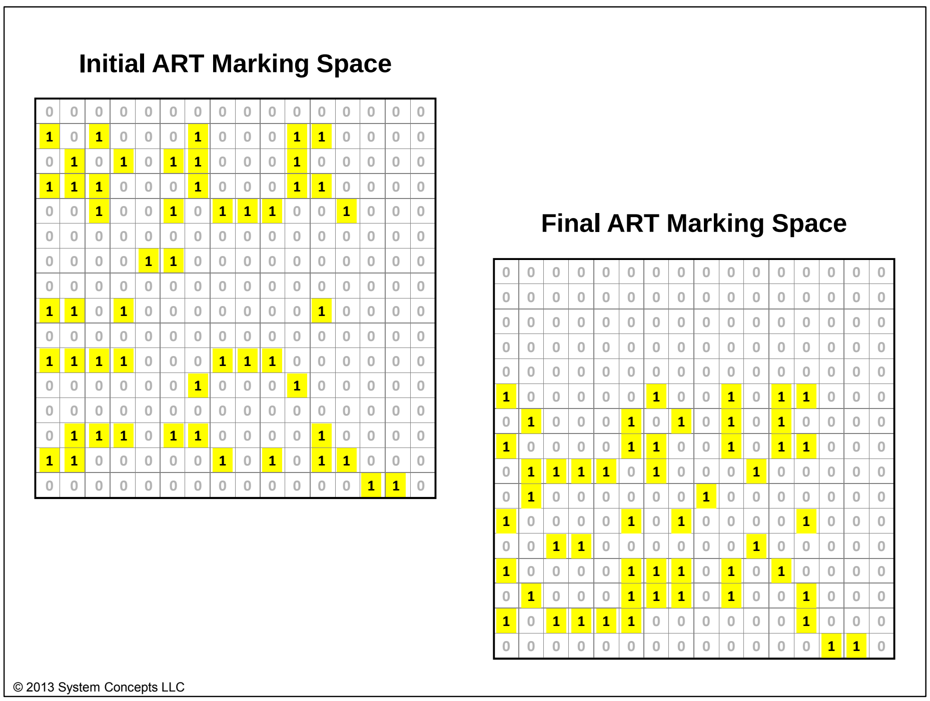


Figure 7. Electric Car ART DSM

The primary system structural consideration in the ART approach is the system organizing relation. The system structural relation is determined by analyzing the natural language system relationship and determining the relational attributes associated with the system organizing relationship. The combination of the natural language relationship and its relational attributes creates the system organizing relation. The natural language relationship of “precedes” is found throughout Stewards DSM published literature. A precedence matrix is also found throughout the DSM literature.

The natural language relationship "precedes" is well understood but it is asymmetric and directional, and needs to be accompanied by a natural language relationship 'succeeds' to provide a concept of task ordering. Task A is the first task, followed by task B and then task C. Task A precedes task B, which precedes task C. But the lack of the 'succeeds' relationship provides a key insight into the DSM method. If you have a set of tasks that are modeled concurrently as a group or task cluster, then you do not need to have the concept of 'succeeds' because everything is executed in a simultaneous fashion. The 'precedes' ordering is then focused on the task clusters, and not the tasks in the clusters. There is a binding relationship between the order of the task, and the order of the task clusters; this is the primary relationship upon which classical DSM focuses.

The group of five empty rows at the top of the electric car can now be understood to create a cluster of tasks that may happen concurrently at the beginning of the project. From an ART 'marking space' point of view, these tasks are not 'connected-to' each other, and should be combined into one task that is 'connected-to' the other tasks. Because the classical DSM approach combines value information and structural information on the same matrix, it is difficult to eliminate these types of system representations. In terms of the classical DSM approach, these five rows, at the top, will never change. The rows will be empty at the beginning of the process, and the rows will be empty at the end of the process. Because the final configuration of these rows is well known before the analysis is performed, these rows contain no information and can be compressed into one row for analysis [Simpson and Simpson, 2012].

The ART DSM approach has a clear separation of system structural representation and system value representation. This more diverse set of representation spaces enables the system ART form to represent a system in more than one manner. An ART DSM approach that combines the tasks that are not connected-to each other will be presented next. In the next ART DSM electric car analysis example, the upper triangular feed-forward marking associated with the N-Squared approach will be used to clearly distinguish between these two examples.

The design activities are represented by a precedence table, and a precedence matrix. The precedence table is shown in Table I, and the precedence matrix is shown in Figure 8.

Table I – DSM Electric Car Precedence Table

Variable (Task)	**Task Description**	**More Sensitive** (Predecessor Tasks)	**Less Sensitive** (Predecessor Tasks)
1	Passenger capacity specifications	None	None
2	Size-aerodynamics	1, 7	3, 11, 12
3	Motor specifications and weight	2, 4, 11	6, 7
4	Total weight	1, 2, 7, 12	3, 11
5	Stored energy requirement	8, 9, 13	3, 6, 10
6	Battery type-energy density	None	None
7	Battery size and weight	5, 6	None
8	Cruising speed specifications	None	None
9	Speed and acceleration performance vs power	2, 4	1, 12

Variable (Task)	**Task Description**	**More Sensitive** (Predecessor Tasks)	**Less Sensitive** (Predecessor Tasks)
10	Acceleration specifications	None	None
11	Speed and acceleration conformance	8, 9, 10	None
12	Structural and suspension design	4	1, 2, 3, 7, 11
13	Range specification	None	None
14	Cost	2, 3, 4, 6, 7, 12	None
15	Consumer demand vs cost	1, 8, 10, 13	None
16	Profit	14, 15	None

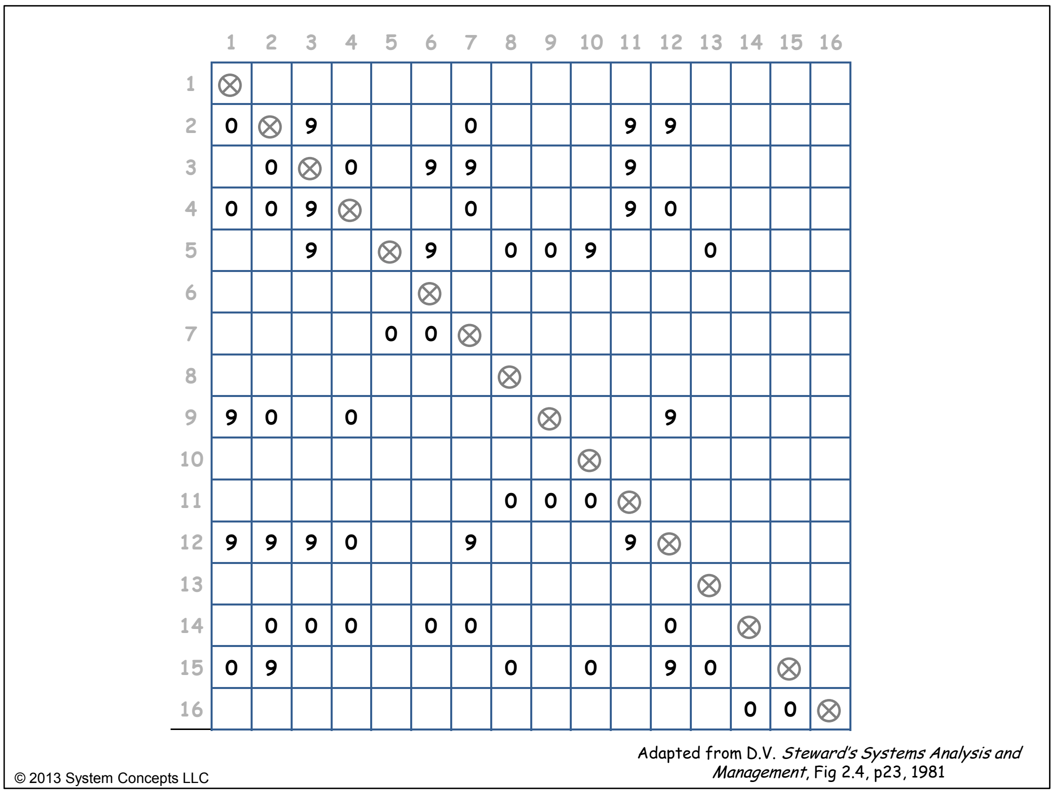


Figure 8. DSM Electric Car Precedence Matrix.

Initially, the ART form of systems structuring and analysis determines the system organizing relationship. Once this relationship is identified, then empirical, observed data from the current system context are used to refine and transform the identified natural language relationship into a natural language relation. The *transformation from relationship to relation is accomplished by identifying the relational attributes associated with the contextual relationship*. In the example of the DSM Electric Car analysis, the candidate natural language relationship is 'precedes.' This relationship is asymmetric and directional, indicating the order of the listed tasks.

Steward's standard DSM method assigns a feedback meaning to marks that are placed *above* the diagonal in the precedence matrix. In this example, the authors use the N-Squared convention of mark placement, wherein marks *below* the matrix diagonal indicate feedback, and marks above the diagonal indicate feed-forward. In addition, information about the quality and/or system impact of a specific task is encoded in the matrix. DSM uses numerals other than zero (0) as an indication of the relative value of each specific task characteristic. As a result, a DSM matrix contains empirical, contextual information, in addition to system structural data.

Further analysis of Steward's 'precedes' (natural language) relationship reveals a dual-level mathematical interpretation of that natural language relationship. Steward provides an analysis of the natural language relationship "precedes" which assigns relational properties of the 'less than or equal to' mathematical operator to the 'precedes' relation, followed by an assignment of relational attributes used to create an 'equivalence operator'. As [Steward, 1981:43-44] specifies:

> "If $x_i \leq x_j$ we say that x_i "precedes" x_j. Note that by definition each x_i precedes itself… "

As a consequence, this dual-level mathematical relation, combined with weak ordering, increases complexity on many different levels.

- One level of the 'precedes' relationship is applied **to the series of blocks** that contain the grouped or clustered tasks.
- The other level of the 'precedes' relationship is applied **to the ordering of tasks within a block**.

This dual-level application of the natural-language relationship, increases the cognitive complexity associated with the classical DSM approach. This specific application of the 'precedes' natural language relationship, creates a situation where empirical data is difficult to collect. If a person has partial knowledge about task sequencing in the process of interest, that person would have a difficult time answering the question, "Please list all tasks that precede themselves." In fact, it is difficult to clearly understand a real world configuration that allows a task to precede itself.

The empirical information associated with task ordering sensitivity is less difficult to gather than the empirical information associated with the task sequencing. However, there are a number of challenging aspects associated with assigning values to the more-sensitive and less-sensitive categories. The first challenge is associated with the global nature of the values. These values must be unchanged for any possible system configuration, if they are to provide a constant structuring metric. The number of configuration permutations makes it difficult to verify the global nature of these sensitivity metrics. The second challenge is the general form that is used to apply these metrics. The order of metric application (or task execution) becomes important for the complete execution chain. To determine a final system metric, the complete task execution order must be evaluated. In systems of any size, this can be computationally overwhelming. The ART process developed by the authors is execution-state independent, and more computationally efficient.

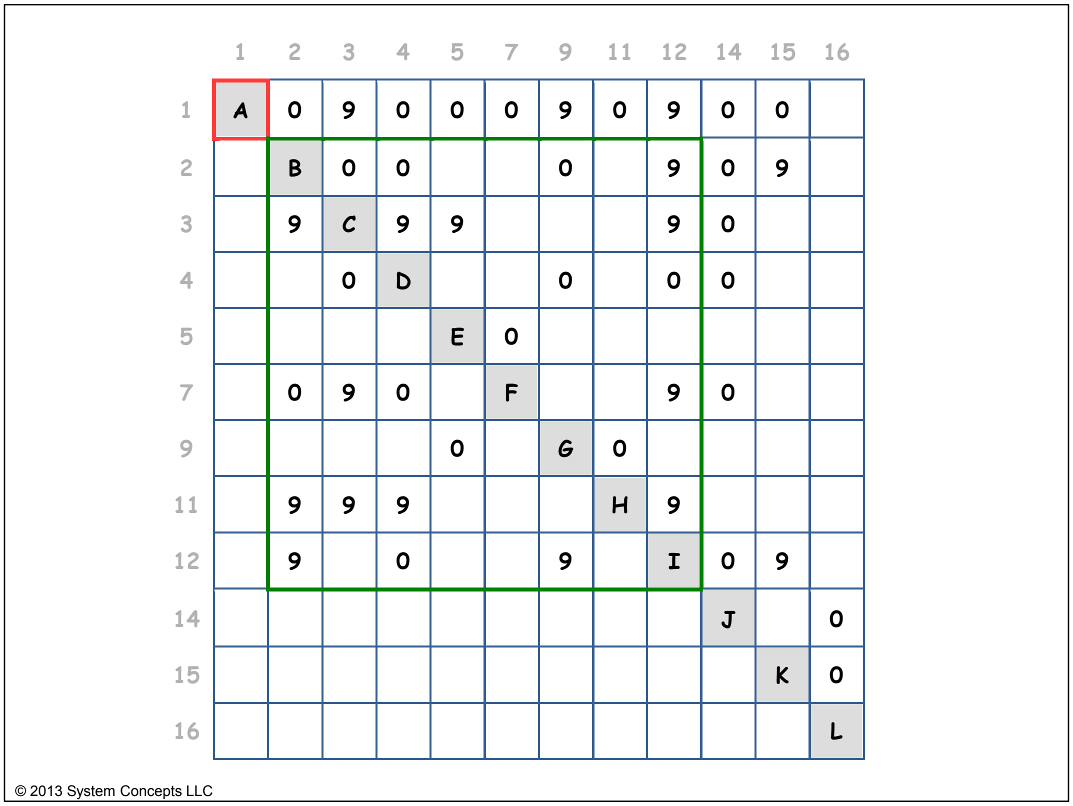
The ordered DSM matrix associated with the Electric Car example is shown in Figure 9.

	1	6	8	10	13	2	3	4	5	7	9	11	12	14	15	16
1	⊗					0		0			9		9		0	
6		⊗					9		9	0				0		
8			⊗						0			0			0	
10				⊗					9			0			0	
13					⊗				0						0	
2						⊗	0	0			0		9	0	9	
3						9	⊗	9	9				9	0		
4							0	⊗			0		0	0		
5									⊗	0						
7						0	9	0		⊗			9	0		
9									0		⊗	0				
11						9	9	9				⊗	9			
12						9		0			9		⊗	0	9	
14														⊗		0
15															⊗	0
16																⊗

Figure 9. Ordered DSM Electric Car Matrix.

The task ordering in Figure 9 may be accomplished by visual inspection of the task description list given in Table 1. Figure 9 shows five (5) tasks that have no predecessors and therefore must all be able to be executed first. These independent first tasks are: Task 1, Task 6, Task 8, Task 10, and Task 13. Because these tasks are independent, the ordering given in Figure 9 is arbitrary. There are a large number of valid initialization sequences for these disconnected tasks. For example, all five tasks could start in a concurrent fashion, or all tasks could start in a sequential fashion with a random ordering of task sequence. Further, there could be many valid task initialization sequences that are composed of two groups: concurrent tasks and sequential tasks. All of these approaches are valid because the tasks are not constrained by the precedence relationship.

The ART DSM approach requires a global system structural relationship. It is clear that the “precedes-succeeds” relationship is useful for ordering local tasks, but may not be the optimum global structuring relationship. As a result, the authors selected the connected-to relationship as the global structuring relationship for this ART DSM example. The ART marking space represents the structure of the system ordered by the global relationship. The ART value space represents the values associated with the connections between local system components, or tasks in this case. Figure 10 presents the tasks from Table 1 in a manner that shows the structural connections between each task. Tasks 1, 6, 8, 10 and 13 have been compressed into one task, marked with an 'A' on the matrix diagonal and placed as the first task in the sequence of connected tasks.



	1	2	3	4	5	7	9	11	12	14	15	16
1	A	0	9	0	0	0	9	0	9	0	0	
2		B	0	0			0		9	0	9	
3		9	C	9	9				9	0		
4			0	D			0		0	0		
5					E	0						
7		0	9	0		F			9	0		
9					0		G	0				
11		9	9	9				H	9			
12		9		0			9		I	0	9	
14										J		0
15											K	0
16												L

Figure 10. Ordered example with compressed block A.

Information from the classical DSM example is mapped to the components of the ART DSM approach. Connections associated with the classical DSM approach are indicated by a number, 0, 5, 9 or an X. The matrix representations that use numbers to indicate a connection, assign a sensitivity weight to these connections. If a zero (0) is used to mark a connection, then this indicates that this is a very sensitive connection. If a nine (9) is used to mark the connection then this indicates an insensitive connection. In any given domain analysis, domain experts are asked to rate the sensitivity of each connection. Domain-specific, empirical data is used in the interpretive model development process of the DSM. A connection assigned the number five (5), is more sensitive than a connection assigned a number seven (7). A connection assigned a number five (5), is less sensitive than a connection assigned the number three (3). The assignment of the values associated with this graded scale, is one of the primary mechanisms used by Steward to integrate aspects of interpretive structural models into basic structural models. In the ART DSM approach, these types of interpretive value scales are located in an ART DSM value space.

The ART DSM marking space is shown in Figure 11. Here, only the presence of a connection (indicated by a one (1)) or the absence of a connection (indicated by a zero (0)) is shown.

	1	2	3	4	5	7	9	11	12	14	15	16
1	A	1	1	1	1	1	1	1	1	1	1	0
2	0	B	1	1	0	0	1	0	1	1	1	0
3	0	1	C	1	1	0	0	0	1	1	0	0
4	0	0	1	D	0	0	1	0	1	1	0	0
5	0	0	0	0	E	1	0	0	0	0	0	0
7	0	1	1	1	0	F	0	0	1	1	0	0
9	0	0	0	0	1	0	G	1	0	0	0	0
11	0	1	1	1	0	0	0	H	1	0	0	0
12	0	1	0	1	0	0	1	0	I	1	1	0
14	0	0	0	0	0	0	0	0	0	J	0	1
15	0	0	0	0	0	0	0	0	0	0	K	1
16	0	0	0	0	0	0	0	0	0	0	0	L

Figure 11. ART Marking Space for Electric Car.

The specific ART DSM values space associated with this example is populated with two value matrices: a global value matrix and a local value matrix. The global value matrix is designed based on the mathematical properties associated with the global structuring relationship, and the basic form of the evolutionary algorithm. A design requirement associated with the evolutionary approach is the presence of a best-fit function. In this case, the best-fit function is a minimization function. Further, the global value space is static and unchanging after the initial value assignment. The transitive nature of the global structuring relationship is also used in the design of the global value matrix. The global value matrix produces a relative scale reading in the search for the best-fit minimum system configuration. The magnitude of the values in the global value matrix and the values local value matrix must be adjusted to produce the designed integrated system value output. The signal from the local matrix must not overwhelm the signal from the global matrix.

The global value matrix for this example is shown in Figure 12. The best-fit function is a minimization function, and the feed-forward section is the upper triangular area. These two criteria require the placement of the lowest value numbers in the upper triangular area of the matrix. The global system relation also provides guidance for the construction of the evolutionary computing rules and procedures. The relation is irreflexive, so a task cannot connect to itself. This fact is represented by the non-numeric values on the diagonal (which are converted to zeros (0) for computation), and a set of evolutionary computation rules that forbid the placement of a connection in a cell along the diagonal. This specific example also provides additional information in the form of initial task identification. Task A is first, and may not be moved. All other tasks are candidates for evaluation and placement in a different task sequence.

	1	2	3	4	5	7	9	11	12	14	15	16
1	A	1	2	3	4	5	6	7	8	9	10	11
2	33	B	3	4	5	6	7	8	9	10	11	12
3	35	34	C	5	6	7	8	9	10	11	12	13
4	37	36	35	D	7	8	9	10	11	12	13	14
5	39	38	37	36	E	9	10	11	12	13	14	15
7	41	40	39	38	37	F	11	12	13	14	15	16
9	43	42	41	40	39	38	G	13	14	15	16	17
11	45	44	43	42	41	40	39	H	15	16	17	18
12	47	46	45	44	43	42	41	40	I	17	18	19
14	49	48	47	46	45	44	43	42	41	J	19	20
15	51	50	49	48	47	46	45	44	43	42	K	21
16	53	52	51	50	49	48	47	46	45	44	43	L

Figure 12. ART Global Value Matrix for Electric Car

The initial local value matrix configuration is a shown in Figure 13. The local value matrix assigns a weighted value to each individual connection. As the structural configuration of the system changes, the local values matrix is adjusted to match the new candidate configuration. The local value matrix is combined with the global value matrix for use in the evolutionary computational approach. The local value matrix has a weighted scale running from one (1) [most sensitive] up to nine (9) [least sensitive]. The computation process has rules for local variable adjustment under the constraints of a specific interpretive domain analysis. The ART DSM analysis is designed to proceed in a series of steps with each step selecting the minimum valued system configuration.

The global system structural relation provides the mathematical properties that are the basis for the value fields in the global value matrix, the relative values in the local value matrix, and rules in the evolutionary computation search process. The ART DSM approach has been shown to be flexible and adaptable to a number of different classical system engineering and system science analytical methods. The authors believe that by placing design emphasis on the natural language system structural relationship, and then transforming that natural language relationship into a mathematic relation, the fundamental system computational properties for a given system can be clearly identified, analyzed, and used as a proven foundation for the solution of large-scale system problems.

	1	2	3	4	5	7	9	11	12	14	15	16
1	A	1	3	1	6	1	3	1	3	1	1	0
2	0	B	1	1	0	0	1	0	3	1	3	0
3	0	3	C	3	3	0	0	0	3	1	0	0
4	0	0	1	D	0	0	1	0	1	1	0	0
5	0	0	0	0	E	1	0	0	0	0	0	0
7	0	1	3	1	0	F	0	0	3	1	0	0
9	0	0	0	0	1	0	G	1	0	0	0	0
11	0	3	3	3	0	0	0	H	3	0	0	0
12	0	3	0	1	0	0	3	0	I	1	3	0
14	0	0	0	0	0	0	0	0	0	J	0	1
15	0	0	0	0	0	0	0	0	0	0	K	1
16	0	0	0	0	0	0	0	0	0	0	0	L

Figure 13. ART Local Value Matrix for Electric Car

The ART DSM electric car design example is a clear example of how these processes and procedures may be applied.

Design Structure Matrix Examples

The ART DSM approach uses the foundations of Basic Structural Modeling and Interpretive Structural Modeling developed by Warfield. Warfield presented clear and detailed connections between system modeling and mathematical structural modeling that established the foundations for his system science work. This foundational work was applied in an interesting manner to the development of DSM methods for managing concurrent engineering tasks. [Eppinger, 1991] referenced Warfield's work *"Binary Matrices in System Modeling."* As shown in Figure 14, Eppinger presented a graphic [Eppinger, 1991:283] with three panels and referred to these graphic representations as directed graphs, or digraphs. The graphics forms presented by Eppinger had little similarity to the referenced work by Warfield, or to standard directed graphs.

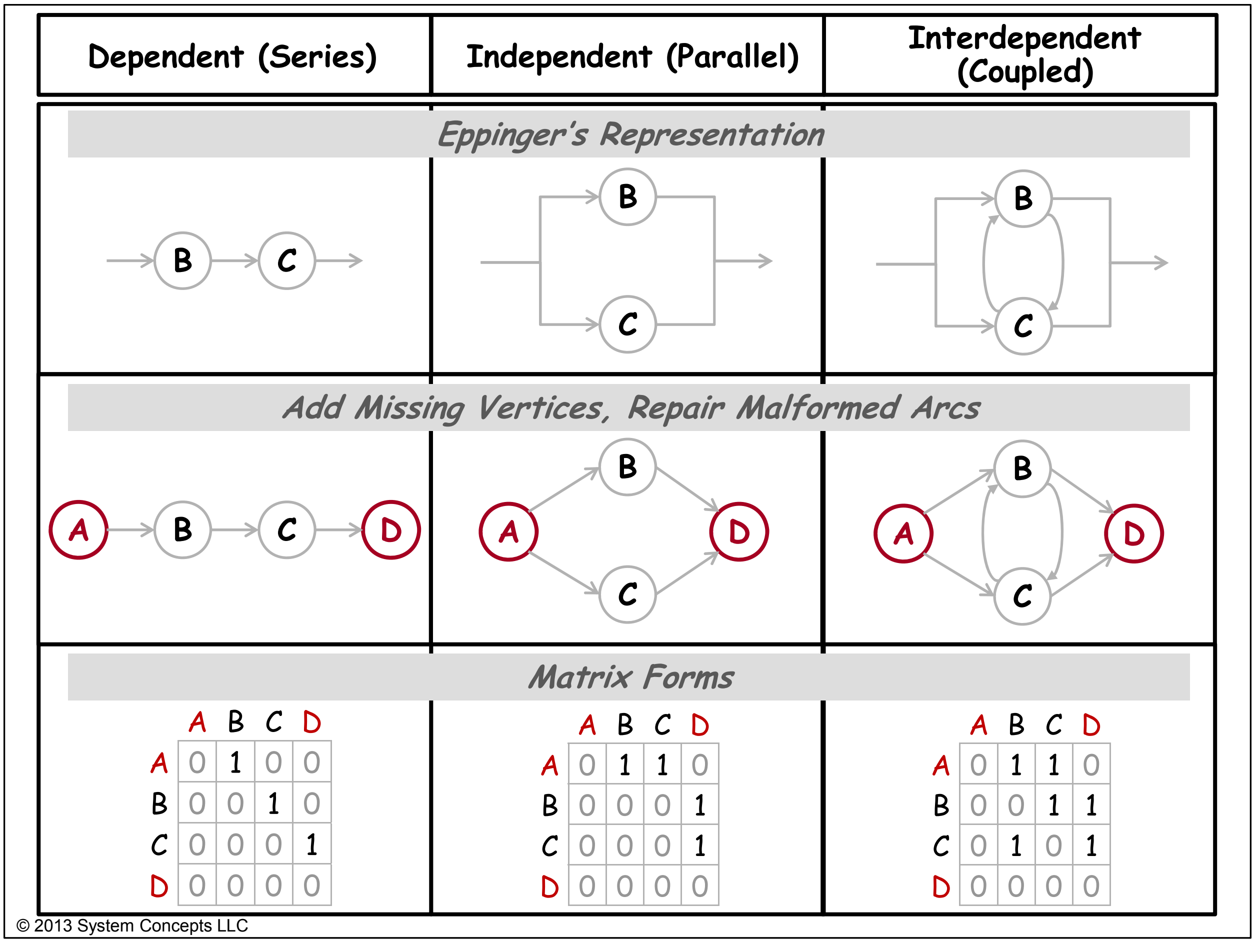


Figure 14. Depiction of "Is-Connected-to" Global Relation

Further analysis of Eppinger's graphic indicates that the figures do not meet the fundamental definition of a directed graph. From Handbook of Discrete and Combinatorial Mathematics: "A directed graph (digraph) consists of [Maurer, 2000:526]:

- a set V, whose elements are called vertices
- a set E, whose elements are called directed edges or arcs, and
- an incidence function that assigns to each edge a tail and a head."

Two directed graph vertices (elements) are missing from the presented graphic representation. These two vertices are added in Figure 14, and the matrix forms corresponding to the connections are presented. The addition of the two missing elements and the proper forming of the arcs, starts to reduce the ambiguity associated with the graphs presented by Eppinger in 1991. Analysis of the Figure 14 matrix forms indicates that all three system configurations are irreflexive. The determination of system symmetry and transitivity would depend on additional empirical data and domain expert interpretation. Warfield presents a process for developing a directed graph in the work referenced by Eppinger. The basis for this process is the identification of a relationship between the elements represented by the directed graph.

The three graphic node configurations presented by Eppinger were evaluated to determine the common relationship among the graphic elements. A detailed review of the first panel, representing a series relationship between the elements of the graph, produces two primary candidate organizing, binary relationships. The first candidate relationship between the elements is “connected-to.” One element is connected to the other element. The second candidate relationship is “precedes-succeeds.” The first element precedes the second element and the second element succeeds the first element. A detailed review of the second panel, representing two independent elements with no interaction between the elements, produces no candidate organizing, binary relationships. A detailed review of the third panel, representing two interdependent elements, with multiple interactions between the elements, produces one candidate organizing, binary relationship. This candidate relationship is “connected-to.”

The process for developing a binary matrix, presented by Warfield, sheds more light on the malformed directed graph presented by Eppinger. Eppinger proposed that all three types of directed graph configurations are allowable into any type of product development DSM analysis. A key issue is the lack of any common, organizing, binary relationship among these three types of node configurations. Once the missing initial and final directed graph elements are added, there appears to be an approach that may create a unified binary relationship that will support the construction and use of binary matrices in DSM system modeling. The essence of this approach is the compression of the disconnected system elements into a single connected system element. After the compression is complete, the connected-to binary relationship applies to all the types of system element configurations.

In the ART DSM electric car example, the five disconnected elements were compressed into one element to support the construction of a well-formed binary matrix. The compression of these elements is a reversible process as described in "Entropy Metrics for System Identification and Analysis" [Simpson and Simpson, 2012].

The original malformed directed graph representation is another indicator that the basic structural forms associated with classical DSM design processes need further analysis and evaluation. It is clear that the ART DSM approach that is organized around a system relationship, identifies each system structural component, assigns a global and local scale value to these elements, and applies evolutionary computation to search for a set of acceptable system configurations is a good candidate tool to use in this analysis and evaluation.

Summary and Conclusions

The use of Abstract Relation Types (ART) in the analysis of system structure and system component clustering has been presented in this paper. Two basic object-clustering definitions, obtained from a literature search, were discussed and evaluated with respect to system structural modeling tasks. System structural modeling techniques presented in this paper are based on Warfield's development of basic structural models and interpretive structural models. The ART approach assigns basic structural system information to the marking space and interpretive structural system information to the value space. The ART analysis approach was applied to a set of classical system engineering examples, with specific emphasis on clustering methods, types and meaning. The process used to identify system relationship attributes and create an ART global structuring relation were also presented and discussed. The primary structuring relationship associated with each example of N-Squared ART and DSM ART were evaluated and discussed in terms of the ART technique. More research is needed to further develop the general ART method and techniques.

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